

One loop back reaction on chaotic inflation

L. R. Abramo* and R. P. Woodard†

Department of Physics, University of Florida, Gainesville, Florida 32611

(Received 18 December 1998; published 15 July 1999)

We extend, for the case of a general scalar potential, the inflaton-graviton Feynman rules recently developed by Iliopoulos *et al.* [Nucl. Phys. **B534**, 419 (1998)]. As an application we compute the leading term, for late comoving times, of the one loop back reaction on the expansion rate for $V(\varphi) = \frac{1}{2}m^2\varphi^2$. This is expressed as the logarithmic time derivative of the scale factor in the coordinate system for which the expectation value of the metric has the form $\langle 0|g_{\mu\nu}(\bar{t}, \vec{x})|0\rangle dx^\mu dx^\nu = -d\bar{t}^2 + a^2(\bar{t})d\vec{x}\cdot d\vec{x}$. This quantity should be a gauge-independent observable. Our result for it agrees exactly with that inferred from the effect previously computed by Mukhanov and co-workers [Phys. Rev. Lett. **78**, 1624 (1997); Phys. Rev. D **56**, 3248 (1997)] using canonical quantization. It is significant that the two calculations were made with completely different schemes for fixing the gauge, and that our computation was done using the standard formalism of covariant quantization. This should settle some of the issues recently raised by Unruh (astro-ph/9802323). [S0556-2821(99)04814-6]

PACS number(s): 04.60.-m, 98.80.Cq

I. INTRODUCTION

We wish to address a controversy which has arisen in the literature of scalar-driven inflation. The dispute concerns the recent claim by Mukhanov and co-workers [1,2] that infrared modes can generate a significant one loop back reaction which reduces the expansion rate over the course of inflation. Unruh [3] has raised a number of serious questions about their methodology and the plausibility of their conclusion.

We begin by summarizing Unruh's objections.

(1) It is difficult to understand how long wavelength modes can affect the local geometry since they should appear spatially constant to a local observer.

(2) To leading order in the long wavelength expansion the mode solutions are all equivalent, locally, to coordinate transformations which can have no effect on local invariants.

(3) The quantization procedure employed by Mukhanov and co-workers is suspect because their dynamical variable is nonzero for only one of the two leading long wavelength solutions. Since this dynamical variable possesses another, independent solution, the corresponding degree of freedom must be unphysical.

(4) What Mukhanov and co-workers refer to as "gauge-independent" quantities are really just the local dynamical variables in a particular gauge.

(5) Mukhanov and co-workers employ an unconventional variation of perturbation theory in which the effective stress-energy tensor of the first order equations renormalizes the zeroth order stress-energy tensor.

(6) The contributions to the metric at second order—the zero mode of which is what Mukhanov and co-workers

computed—depend upon the gauge chosen for expressing the first order terms.

Without wishing to criticize good people who addressed an important issue at the extreme limit of their formalism's applicability, we must admit to a certain sympathy for Unruh's methodological objections. One of the present authors also had difficulty understanding the work of Mukhanov and co-workers on account of objections (4)–(6).¹ However, we believe the physics of what they did is correct, and that is the point of this paper.

After fixing notation about the perturbative background in Sec. II we comment in Sec. III on the physics of the process and we partially address Unruh's objections (1)–(3). The remainder of the paper is devoted to checking the calculation of Mukhanov and co-workers, in a completely different gauge, using the standard formalism of covariant quantum field theory. The Feynman rules are given in Sec. IV. These were lifted from a recent paper by Iliopoulos, Tomaras, Tsamis, and Woodard [4], which we have extended so that the propagators can be computed (as mode sums) for a general scalar potential. Section V attaches the external lines (which are retarded propagators in Schwinger's formalism [5]) needed to convert the amputated one-point functions into the expectation values of the metric and the scalar. We also explain how these expectation values are used to compute physical observables which measure the cosmological expansion rate and the evolution of the scalar. In Sec. VI we give the procedure used for isolating the leading contribution to each propagator from superadiabatically amplified modes at late comoving times. This is the chief physical approximation of the paper. The amputated one-point functions are

*Present address: Theoretische Physik, Ludwig Maximilians Universität, Theresienstrasse 37, D-80333 München, Germany. Email address: abramo@theorie.physik.uni-muenchen.de

†Email address: woodard@phys.ufl.edu

¹Other confusing points were the characterization of what is obviously a one loop effect as "classical" and the attribution of this effect to an instability in the classical energy functional (which is actually stable).

computed in Sec. VII and processed to give the two physical observables. Section VIII summarizes the various results.

II. THE PERTURBATIVE BACKGROUND

The system under study is that of general relativity with a general minimally coupled scalar

$$\mathcal{L} = \frac{1}{16\pi G} R \sqrt{-g} - \frac{1}{2} \partial_\mu \varphi \partial_\nu \varphi g^{\mu\nu} \sqrt{-g} - V(\varphi) \sqrt{-g}. \quad (1)$$

This section concerns the homogeneous and isotropic backgrounds g_0 and φ_0 about which perturbation theory will be formulated. Three classes of identities turn out to be interesting for our purposes: (1) those which are exact and valid for any potential $V(\varphi)$, (2) those which are valid in the slow roll approximation but still for any potential, and (3) those which are valid for the slow roll approximation with the potential $V(\varphi) = \frac{1}{2} m^2 \varphi^2$. We shall develop them in this order, identifying the point at which each further specialization and approximation is made.

Among the exact identities is the relation between comoving and conformal coordinates:

$$ds_0^2 = -dt^2 + a_0^2(t) d\vec{x} \cdot d\vec{x} = \Omega^2(\eta) \{-d\eta^2 + d\vec{x} \cdot d\vec{x}\}. \quad (2)$$

This implies

$$dt = \Omega d\eta, \quad a_0(t) = \Omega(\eta). \quad (3)$$

The Hubble “constant” is the logarithmic comoving time derivative of the background scale factor

$$H \equiv \frac{\dot{a}_0}{a_0} = \frac{\Omega'}{\Omega^2}, \quad (4)$$

where a dot denotes differentiation with respect to (background) comoving time and a prime stands for differentiation with respect to conformal time.

Two of Einstein’s equations are nontrivial in this background:

$$3H^2 = \frac{1}{2} \kappa^2 \left\{ \frac{1}{2} \dot{\varphi}_0^2 + V(\varphi_0) \right\}, \quad (5)$$

$$-2\dot{H} - 3H^2 = \frac{1}{2} \kappa^2 \left\{ \frac{1}{2} \dot{\varphi}_0^2 - V(\varphi_0) \right\}, \quad (6)$$

where $\kappa^2 \equiv 16\pi G$ is the loop counting parameter of perturbative quantum gravity. One can use the two Einstein equations to derive the scalar equation of motion

$$\ddot{\varphi}_0 + 3H\dot{\varphi}_0 + V_{,\varphi}(\varphi_0) = 0, \quad (7)$$

where $V_{,\varphi} \equiv \partial V / \partial \varphi$. One can also invert the Einstein equations to solve for the Hubble constant and its first derivative

$$H^2 = \frac{1}{6} \kappa^2 \left\{ \frac{1}{2} \dot{\varphi}_0^2 + V(\varphi_0) \right\}, \quad (8)$$

$$\dot{H} = -\frac{1}{4} \kappa^2 \dot{\varphi}_0^2. \quad (9)$$

Sometimes it is more convenient to write the scalar quantities in terms of the Hubble constant and its derivative

$$\dot{\varphi}_0^2 = -\frac{4}{\kappa^2} \dot{H}, \quad (10)$$

$$V(\varphi_0) = \frac{2}{\kappa^2} (\dot{H} + 3H^2). \quad (11)$$

At other times one wants to express the scalar quantities using the conformal factor Ω

$$\dot{\varphi}_0^2 = \frac{4}{\kappa^2} \frac{1}{\Omega^2} \left\{ -\frac{\Omega''}{\Omega} + 2 \left(\frac{\Omega'}{\Omega} \right)^2 \right\}, \quad (12)$$

$$V(\varphi_0) = \frac{2}{\kappa^2} \frac{1}{\Omega^2} \left\{ \frac{\Omega''}{\Omega} + \left(\frac{\Omega'}{\Omega} \right)^2 \right\}. \quad (13)$$

The conformal time derivative of the scalar $\varphi'_0 = \Omega \dot{\varphi}_0$ is also useful,

$$\varphi_0'^2 = \frac{4}{\kappa^2} \left\{ -\frac{\Omega''}{\Omega} + 2 \left(\frac{\Omega'}{\Omega} \right)^2 \right\}. \quad (14)$$

Successful models of inflation require the following two conditions which define the *slow roll approximation*

$$|\ddot{\varphi}_0| \ll H |\dot{\varphi}_0|, \quad (15)$$

$$\dot{\varphi}_0^2 \ll V(\varphi_0). \quad (16)$$

It follows that there are two small parameters. Although these are traditionally expressed as ratios of the potential and its derivatives the more useful quantities for our work are ratios of the Hubble constant and its derivatives

$$\frac{-\dot{H}}{H^2} \ll 1, \quad \frac{|\ddot{H}|}{-H\dot{H}} \ll 1. \quad (17)$$

For models of interest to us the rightmost of these parameters is negligible with respect to the leftmost one. We shall also assume that the derivative of the scalar is negative,

$$\varphi'_0 = -\frac{2}{\kappa} \Omega \sqrt{-\dot{H}}. \quad (18)$$

The slow roll approximation gives useful expansions for simple calculus operations. For example, ratios of derivatives of the field are

$$\frac{\varphi_0''}{\varphi_0'} = H\Omega \left(1 + \frac{1}{2} \frac{\ddot{H}}{H\dot{H}} \right), \quad (19)$$

$$\frac{\varphi_0'''}{\varphi_0'} = 2H^2\Omega^2 \left(1 + \frac{\dot{H}}{2H^2} + \dots \right). \quad (20)$$

Successive partial integration also defines useful slow roll expansions

$$\int dt H^\alpha \Omega^\beta = \frac{1}{\beta} H^{\alpha-1} \Omega^\beta \left\{ 1 + \frac{(\alpha-1)}{\beta} \left(\frac{-\dot{H}}{H^2} \right) + \dots \right\}, \quad (21)$$

$$\int dt H^\alpha = \frac{1}{\alpha+1} \frac{H^{\alpha+1}}{\dot{H}} \left\{ 1 + \frac{1}{\alpha+2} \frac{H\ddot{H}}{\dot{H}^2} + \dots \right\}. \quad (22)$$

In discussing the physical significance of their result Mukhanov, Abramo, and Brandenberger specialized to the simplest potential for chaotic inflation [6]

$$V(\varphi) \rightarrow \frac{1}{2} m^2 \varphi^2. \quad (23)$$

In the slow roll approximation with this potential one can solve explicitly for the scalar's evolution

$$\varphi_0(t) = \varphi_i - \frac{2}{\sqrt{3}} \frac{m}{\kappa} t. \quad (24)$$

The interesting geometrical quantities have the following expressions in terms of $\varphi_0(t)$:

$$\dot{H} \approx -\frac{1}{3} m^2, \quad (25)$$

$$H \approx \frac{1}{\sqrt{12}} \kappa m \varphi_0(t). \quad (26)$$

Note that $\ddot{H} \approx 0$ for this potential, so only one of the slow roll parameters is nonzero. The slow roll approximation implies that the initial value of the scalar field is much larger than the Planck mass:

$$\varphi_i \gg \frac{1}{\kappa}. \quad (27)$$

Inflation ends in this model when $\varphi_0(t) \sim \kappa^{-1}$.

III. PHYSICAL COMMENTS

The physical mechanism behind what Mukhanov, Abramo, and Brandenberger [1,2] have found for scalar-driven inflation is roughly the same as that studied previously by Tsamis and Woodard [7] in the context of inflation caused by a bare cosmological constant. There is such a simple physical model for what is going on that we would be derelict in our duty of explication not to present it. Formalists should rest assured that this is merely a qualitative description of phenomena whose reality has already been established by computing what should be invariant observables in the standard formalism of covariant quantization.

Owing to the rapid expansion of spacetime and the special properties of the dynamical quanta involved,² there is a vast enhancement of the 0-point energy which the uncertainty principle requires to be present in each dynamical degree of freedom. This is the phenomenon of *superadiabatic amplification*, first studied by Grishchuk [8]. A simple picture for it is that virtual pairs with wavelengths comparable to the horizon can become trapped in the expansion of spacetime and not be able to recombine.

Superadiabatic amplification is not a large effect by itself. Although the total energy contained in infrared modes increases quite rapidly, the corresponding expansion of the three-volume keeps the energy *density* constant for pure de Sitter expansion.³ For this background it is simple to show that there is only about one extra infrared quantum per Hubble volume. The interesting, secular effect derives from the gravitational interaction between these quanta. As each virtual pair is pulled apart, its gravitational potentials fill the intervening space. These remain to add with those of the next pair. Even though the 0-point energy stays constant, the induced gravitational potential increases. It is the interaction energy between this and the 0-point energy, and between the gravitational potentials themselves, which gives the effect.

In the purely gravitational model of Tsamis and Woodard, linearized gravitons can only induce gravitational potentials at second order in the weak field expansion. Since superadiabatic amplification is a one loop effect this means that the secular back reaction comes at two loop order. When inflation is driven by a scalar field its quanta can induce gravitational potentials even at linearized order in the weak field expansion. This is why Mukhanov, Abramo, and Brandenberger were able to follow what is essentially the same physical process with a vastly simpler one loop calculation.

Either way, the effect is to slow inflation because gravity is attractive. Since gravity is also a weak interaction, even for grand unified theory (GUT) scale inflation, the process requires an enormous amount of time before it can become significant. A direct consequence is that the equation of state of the induced stress tensor must be approximately that of negative vacuum energy. To see this consider the relation implied by conservation between the induced energy density $\rho(t)$ and the induced pressure $p(t)$

$$\dot{\rho} = -3H(\rho + p). \quad (28)$$

Since the accumulation of a significant effect requires many Hubble times $|\dot{\rho}| \ll H|\rho|$ and it must be that $p(t)$ nearly cancels $\rho(t)$.

²These properties are (1) effective masslessness on the Hubble scale and (2) the absence of classical conformal invariance.

³For certain models of scalar-driven inflation the infrared energy density can grow as the scalar rolls. This is what seems to distinguish those scalar potentials for which there is a one loop effect from those for which there is not. The two loop effect of pure gravity—and presumably also gravity with scalars—does not depend upon such growth.

A sometimes confusing point is that one does not require the complete theory of quantum gravity in order to study an infrared process such as this. As long as spurious time dependence is not injected through the ultraviolet regularization, the late time back-reaction is dominated by ultraviolet finite, nonlocal terms whose form is entirely controlled by the low energy limiting theory. This theory must be general relativity, with the possible addition of some light scalars. It is worth commenting that infrared phenomena can always be studied using the low energy effective theory. This is why Bloch and Nordsieck [9] were able to resolve the infrared problem of QED before the theory's renormalizability was suspected. It is also why Weinberg [10] was able to achieve a similar resolution for quantum general relativity with zero cosmological constant. And it is why Feinberg and Sucher [11] were able to compute the long range force due to neutrino exchange using Fermi theory. More recently Donoghue [12] has been working along the same lines for quantum gravity with zero cosmological constant.

We emphasize that the process is causal, in spite of its close association with modes whose wavelengths have redshifted beyond the horizon. This emerges most clearly in the two loop computation of Tsamis and Woodard where the effect derives from integrating interaction vertices over the past light cone of the point at which the expansion rate is being measured. Because gravitons are massless these interactions superpose coherently. Because gravitons are not conformally invariant they reflect the enormous physical volume of the past light cone rather than its minuscule conformal volume. The growth in the back reaction is directly attributable to the fact that the invariant volume of the past light cone increases without bound as one observes at later and later times.

Causality is also built into the work of Mukhanov, Abramo, and Brandenberger through their use of the Heisenberg field operators. The equations of motion for these are simply operator realizations of the causal field equations of classical general relativity. In a local gauge one can express an operator at the spacetime point $(t, \vec{x})$ entirely in terms of the operators and their time derivatives on that part of the initial value surface which lies on or within the past light cone of $(t, \vec{x})$. There might be some dispute about what this means nonperturbatively, where the quantum metric can have a significant impact on the light cone, but it makes perfect sense in the perturbative regime under study. The time dependence Mukhanov, Abramo, and Brandenberger obtain derives in part from the continual redshift of new modes from ultraviolet to infrared but mostly from the growth of the infrared mode functions which can occur in some (but not all) inflationary backgrounds.

We turn now to issues (1)–(3) listed in our Introduction. Regarding the first objection, it is relevant to note that while long wavelength modes indeed appear spatially constant to a local observer, so too does the cosmological expansion rate of an inflating universe. Therefore the causative agent and its purported effect are commensurate: the homogeneous expansion rate of the classical background is being screened by the quantum dynamics of modes which would seem homogeneous to a local observer.

Viewed from the perspective of obtaining a long period of inflation it is rather local, short wavelength phenomena that ought to be regarded with suspicion. Without severe fine-tuning the natural duration of any process mediated by short wavelength quanta must be the Hubble time or less. It seems reasonable to conclude that a mechanism for screening the cosmological constant must also end inflation if nothing else does the job first. But inflation has to persist for many Hubble times in order to explain the large scale smoothness of the observed universe. Note that infrared, long wavelength phenomena can require much longer to produce a significant effect because they can act by coherently superposing an inherently weak interaction over the past light cone. It is only an enormous expansion of the invariant volume contained in the past light cone that can compensate for the weakness of gravitational self-interactions.

Finally, one must distinguish a local observation of a truly homogeneous expansion from the sort of local expansion that would be produced by spatial inhomogeneities in the vacuum energy. Many people believe that whatever is suppressing the former must also suppress the latter. We do not share this view. Experiment is sadly unable to decide the matter but it seems to us that a local fluctuation which created a large enough region of negative $\rho + 3p$ should result in that region beginning to undergo inflation. We believe that known physical principles already suffice to explain why such fluctuations are rare in the observed universe [13]. Were it otherwise one would not be able to make conventional models of inflation agree with observation by the unaesthetic device of fine-tuning the bare cosmological constant which is, be it noted, spatially homogeneous.

Two points are relevant for the second objection. First, the “stress tensor” of gravitational perturbation theory is not an invariant, or even a scalar. So the fact that infrared mode solutions, to leading order for small wave number, are coordinate transformations does not mean they necessarily have no effect. Second, it is only for strictly zero wave number that infrared modes are pure gauge. Superadiabatic amplification allows these modes to carry nonzero energy and pressure in spite of their extreme redshift. The proper way to determine their effect is by computing the metric's response at second order and then addressing the gauge issue of what this response means physically. We believe this is what Mukhanov, Abramo, and Brandenberger did, although perhaps not as transparently as one might wish. To check their result we made what ought to be the same computation in a completely different gauge and using the standard formalism of covariant quantization, and we got the same answer.

This is the right point to comment on the gauge issue, which was also raised extensively by Unruh. What both we and Mukhanov and co-workers computed was the expectation value of the metric in the presence of a particular state and in a fixed gauge. There is no doubt that this quantity depends upon the gauge in which the computation was done. It is important to realize that a quantity is not automatically devoid of physical import by virtue of being gauge dependent. It can still contain useful physical information which can be separated from the unphysical, gauge-dependent part.

Examples of this abound in quantum field theory. The most straightforward is the way in which gauge dependent Green's functions can be processed, using Lehmann-Symanzik-Zimmermann (LSZ) reduction, to give gauge independent, on-shell scattering amplitudes. (This is discussed in any standard text on quantum field theory, for example that by Peskin and Schroeder [14].) One does not even have to consider products of field operators. There is an elegant formalism, due to DeWitt [15], in which the S matrix is obtained from the in-out matrix element of the dynamical variable in the presence of a general scattering state.

So the expectation value of the metric contains valid physical information; the question is how to extract it. Our technique exploits the special property of the initial state of being homogeneous and isotropic. This means that a comoving coordinate system exists for which⁴

$$\langle 0 | g_{\mu\nu}(\bar{t}, \vec{x}) dx^\mu dx^\nu | 0 \rangle = -d\bar{t}^2 + a^2(\bar{t}) d\vec{x} \cdot d\vec{x}. \quad (29)$$

Our observable is the logarithmic time derivative of the scale factor in this coordinate system

$$H_{\text{eff}}(\bar{t}) \equiv \frac{1}{a(\bar{t})} \frac{da(\bar{t})}{d\bar{t}}. \quad (30)$$

One can investigate how this quantity changes under a variation of the gauge fixing functional and the result is that it does not change [7]. This would seem to be the analogue of DeWitt's theorem about the gauge independence of the on-shell S matrix. Of course the absence of gauge dependence does not automatically endow a quantity with physical import. We interpret $H_{\text{eff}}(\bar{t})$ as the expansion rate a local observer would measure in the presence of state $|0\rangle$. It certainly has this meaning in the classical limit but we are willing to entertain dissident views.

We come finally to Unruh's doubts about the formalism of Mukhanov, Feldman, and Brandenberger [16]. His argument is based on the long wavelength solution [3] he found for the linearized Newtonian potential, which we shall call $n(x)$. In our notation this quantity corresponds to the following invariant element:

$$g_{\mu\nu} dx^\mu dx^\nu = \Omega^2 \{ -(1 + 2\Omega^{-1}n) d\eta^2 + (1 - 2\Omega^{-1}n) d\vec{x} \cdot d\vec{x} \}. \quad (31)$$

When the linearized g_{0i} equations are used to eliminate the scalar field the linearized g_{00} equation becomes

⁴Although homogeneity and isotropy follow trivially in the perturbative formalism we employ, the potential for nonperturbative symmetry breaking should be noted.

$$n'' - \nabla^2 n - 2 \frac{\varphi_0''}{\varphi_0'} n' + \left(\frac{\Omega''}{\Omega} - 2 \frac{\Omega'^2}{\Omega^2} \right) n = 0. \quad (32)$$

In the limit that the ∇^2 term can be neglected Unruh obtains the following independent solutions:

$$n_1 = \Omega - \frac{\Omega'}{\Omega^2} \int_{-\infty}^{\eta} d\bar{\eta} \Omega^2(\bar{\eta}), \quad (33)$$

$$n_2 = \frac{\Omega'}{\Omega^2}. \quad (34)$$

Unruh's problem concerns what happens when n_1 and n_2 are substituted into the dynamical variable used by Mukhanov, Feldman, and Brandenberger

$$v = \frac{4}{\kappa^2} \frac{1}{\varphi_0'} \left\{ n' + \left(-\frac{\Omega''}{\Omega'} + 2 \frac{\Omega'}{\Omega} \right) n \right\}. \quad (35)$$

Although n_1 produces a reasonable function,

$$v_1 = \frac{4}{\kappa^2} \frac{1}{\varphi_0'} \left\{ -\frac{\Omega\Omega''}{\Omega'} + 2\Omega' \right\} = \frac{\Omega^2}{\Omega'} \varphi_0', \quad (36)$$

the n_2 solution gives $v=0$. This is disturbing because the formalism of Mukhanov, Feldman, and Brandenberger quantizes v as a scalar field which obeys the following second order equation

$$v'' - \nabla^2 v - \frac{v_1''}{v_1} v = 0. \quad (37)$$

In the limit that the ∇^2 term can be neglected one finds that $v=v_1$ is indeed a solution as is:

$$v_2(\eta) = v_1(\eta) \int_{\eta}^0 \frac{d\bar{\eta}}{v_1^2(\bar{\eta})}. \quad (38)$$

Since this second solution does not correspond to any combination of Unruh's two long wavelength solutions he concludes that it must be unphysical and that the formalism is therefore suspect.

In fact neither the v_2 solution nor the n_2 solution is unphysical, they simply correspond to different orders in the long wavelength expansion.⁵ At fixed, nonzero wave number $\vec{k}$ one can express the two independent solutions for the Newtonian potential as power series in k^2 . The zeroth terms in these two series are Unruh's solutions $n_1(\eta)$ and $n_2(\eta)$. However, one should really include some higher order terms as well since the physical relevance of the solutions is for

⁵Since the completion of this paper we have learned that the relation between the v 's and the n 's was previously clarified by Kodama and Hamazaki [17]. It should also be noted that Unruh has independently withdrawn the claim of an error in the formalism of Mukhanov, Feldman, and Brandenberger.

modes with small but nonzero wave number.⁶ The first order correction to n_2 can be expressed using an advanced Green's function

$$N_2(\eta, k) = n_2(\eta) + \frac{4k^2}{\kappa^2} \int_{\eta}^0 d\bar{\eta} \{ n_1(\eta) n_2(\bar{\eta}) - n_2(\eta) n_1(\bar{\eta}) \} \frac{n_2(\bar{\eta})}{(\varphi_0'(\bar{\eta}))^2} + O(k^4). \quad (39)$$

When it is substituted for $n(x)$ in Eq. (35) the result is

$$v|_{n=N_2} = \frac{4k^2}{\kappa^2} v_2(\eta) + O(k^4). \quad (40)$$

So the solution sets of the two variables are in one-to-one correspondence and there is no obvious problem with the formalism of Mukhanov, Feldman, and Brandenberger.

At the price of specializing to power law inflation one can even see how the n and v solution sets relate to all orders. For $\Omega = (\eta_0/\eta)^{s/(s-1)}$ the two mode solutions for the Newtonian potential are proportional to Bessel functions of order $\mu = \frac{1}{2} + 1/(s-1)$. Unruh's solutions are the zeroth order terms in the power series expansions of the following:

$$N_1(\eta, k) = \frac{\Gamma(1-\mu)}{s+1} \left(\frac{k\eta_0}{2} \right)^{\mu} \sqrt{\frac{\eta_0}{\eta}} J_{-\mu}(k\eta), \quad (41)$$

$$N_2(\eta, k) = -\frac{s}{s-1} \Gamma(1+\mu) \left(\frac{k\eta_0}{2} \right)^{-\mu} \frac{1}{\sqrt{\eta_0\eta}} J_{\mu}(k\eta). \quad (42)$$

One can easily verify that the solutions to Eq. (37) which make contact with v_1 and v_2 are

$$V_1(\eta, k) = \frac{2}{\kappa} \frac{\Gamma(-\mu)}{\sqrt{s}} \left(\frac{k\eta_0}{2} \right)^{\mu+1} \sqrt{\frac{\eta}{\eta_0}} J_{-\mu-1}(k\eta), \quad (43)$$

$$V_2(\eta, k) = -\frac{\kappa}{4} \sqrt{s} \Gamma(1+\mu) \left(\frac{k\eta_0}{2} \right)^{-\mu-1} \sqrt{\eta_0\eta} J_{\mu+1}(k\eta). \quad (44)$$

For power law inflation the relation (35) between n and v is recognizable as the recursion relation which produces $-J_{\mu+1}$ from J_{μ} (and $+J_{-\mu-1}$ from $J_{-\mu}$)

⁶There are a lot more of these. The number of zero modes is constant in time whereas the inflationary redshift eventually makes the physical wave number ($\Omega^{-1}\vec{k}$) of any mode small. Further, as Unruh pointed out, the $\vec{k}=0$ system is degenerate in that its two solutions become unphysical on account of being exactly coordinate transformations. The one physical solution is paradoxically absent from the $\vec{k}\neq 0$ system. It appears because the g_{0i} equation is automatically satisfied for $\vec{k}=0$ and therefore fails to relate the zero modes of the Newtonian potential and the scalar [3].

$$v \rightarrow -\frac{2}{\kappa} \frac{s-1}{\sqrt{s}} \sqrt{\eta} \left\{ [\sqrt{\eta}n]' - \frac{\mu}{\eta} [\sqrt{\eta}n] \right\} \quad (45)$$

and the V_i 's descend from the N_i 's as follows:

$$V_1(\eta, k) = -\frac{2}{\kappa} \frac{s-1}{\sqrt{s}} \sqrt{\eta} \times \left\{ [\sqrt{\eta}N_1(\eta, k)]' - \frac{\mu}{\eta} [\sqrt{\eta}N_1(\eta, k)] \right\}, \quad (46)$$

$$V_2(\eta, k) = \frac{\kappa^2}{4k^2} \times -\frac{2}{\kappa} \frac{s-1}{\sqrt{s}} \sqrt{\eta} \times \left\{ [\sqrt{\eta}N_2(\eta, k)]' - \frac{\mu}{\eta} [\sqrt{\eta}N_2(\eta, k)] \right\}. \quad (47)$$

IV. FEYNMAN RULES

The purpose of this section is to give the Feynman rules for the general inflaton-graviton action (1). We have mostly borrowed these from a recent paper by Iliopoulos, Tomaras, Tsamis, and Woodard [4]. The one exception concerns the issue of mixing between the scalar and the 00 component of the graviton field. Iliopoulos *et al.* were only able to solve the system for a class of backgrounds including those of power law inflation, but not the power law backgrounds typical of chaotic inflation. We have achieved a general solution. One should also note that all formulas given in this section are exact. We have made neither the slow roll approximation nor have we specialized to the case of a quadratic potential. Of course, that will be necessary in order to convert the formal mode sums into explicit results, but the task of making these approximations has been postponed to the end of Sec. V.

Our quantum fields are the scalar ϕ and the conformally rescaled pseudograviton $\psi_{\mu\nu}$

$$\phi \equiv \varphi_0 + \phi, \quad (48)$$

$$g_{\mu\nu} \equiv \Omega^2(\eta_{\mu\nu} + \kappa\psi_{\mu\nu}) \equiv \Omega^2 \tilde{g}_{\mu\nu}. \quad (49)$$

It should be noted that cosmologists typically restrict the word “graviton” to that part of the metric which interpolates dynamical spin two quanta at linearized order. Adhering to this convention would be terrifically cumbersome in the context of Becchi-Rouet-Stora (BRS) quantization beyond linearized order. Our “pseudograviton” also includes degrees of freedom which are constrained or pure gauge. This is the standard usage in particle theory, cf. the “photon” field, propagator, and interactions of QED and the “gluon” field, propagator, and interactions of QCD [14]. We shall try to avoid misunderstandings, without over-burdening the notation, by following the convention of Iliopoulos *et al.* who parenthesized the word “general” before “pseudograviton.”

As usual, (general) pseudograviton indices are raised and lowered with the Lorentz metric. After many tedious partial integrations the invariant Lagrangian can be written as a total derivative plus the following:

$$\begin{aligned}\mathcal{L}_{\text{inv}} = & \sqrt{-\tilde{g}} \tilde{g}^{\alpha\beta} \tilde{g}^{\rho\sigma} \tilde{g}^{\mu\nu} \left\{ \frac{1}{2} \psi_{\alpha\rho,\mu} \psi_{\nu\sigma,\beta} - \frac{1}{2} \psi_{\alpha\beta,\rho} \psi_{\sigma\mu,\nu} \right. \\ & + \frac{1}{4} \psi_{\alpha\beta,\rho} \psi_{\mu\nu,\sigma} - \frac{1}{4} \psi_{\alpha\rho,\mu} \psi_{\beta\sigma,\nu} \left. \right\} \Omega^2 \\ & - \frac{1}{2} \sqrt{-\tilde{g}} \tilde{g}^{\rho\sigma} \tilde{g}^{\mu\nu} \psi_{\rho\sigma,\mu} \psi_{\nu}^{\alpha} (\Omega^2)_{,\alpha} - \Omega^2 \varphi_0' \phi_{,\mu} \tilde{g}^{0\mu} \sqrt{-\tilde{g}} \\ & - \frac{1}{2} \Omega^2 \phi_{,\mu} \phi_{,\nu} \tilde{g}^{\mu\nu} \sqrt{-\tilde{g}} \\ & - \sum_{n=1}^{\infty} \frac{1}{n!} \frac{\partial^n V(\varphi_0)}{\partial \varphi^n} \Omega^4 \phi^n \sqrt{-\tilde{g}},\end{aligned}\quad (50)$$

where a comma denotes differentiation. Gauge fixing is accomplished by adding a gauge fixing functional and the corresponding ghost action to obtain the BRS Lagrangian

$$\mathcal{L}_{\text{BRS}} = \mathcal{L}_{\text{inv}} - \frac{1}{2} \eta^{\mu\nu} F_{\mu} F_{\nu} - \Omega \bar{\omega}^{\mu} \delta F_{\mu}. \quad (51)$$

The symbol δF_{μ} represents the variation of the gauge fixing functional under an infinitesimal diffeomorphism parametrized by the ghost field ω_{μ} . We will follow Iliopoulos *et al.* in our choice of gauge fixing functional:

$$F_{\mu} = \Omega \left(\psi_{\mu,\nu}^{\nu} - \frac{1}{2} \psi_{,\mu} - 2 \frac{\Omega'}{\Omega} \psi_{\mu 0} + \eta_{\mu 0} \kappa \varphi_0' \phi \right). \quad (52)$$

A great advantage of this gauge is that it decouples the tensor structure of the propagators from their dependence on space-time. The propagator becomes a small number of constant tensors multiplying only three different types of mode sums. Another advantage is that the limit $\Omega \rightarrow 1$ takes this gauge to one of the standard gauges of flat space, which often provides a useful correspondence check.

With a few more partial integrations the terms quadratic in the various quantum fields can be reduced to the following form:

$$\begin{aligned}\mathcal{L}_{\text{BRS}}^{(2)} = & \frac{1}{2} \psi^{\mu\nu} D_{\mu\nu}{}^{\rho\sigma} \psi_{\rho\sigma} + \psi^{\mu\nu} \Omega [-\kappa \varphi_0'' t_{\mu} t_{\nu}] \Omega \phi \\ & + \frac{1}{2} \phi \Omega \left[\partial^2 + \frac{1}{4} \kappa^2 \varphi_0'^2 + \frac{\varphi_0'''}{\varphi_0'} \right] \Omega \phi \\ & + \bar{\omega}^{\mu} [\bar{\delta}_{\mu}^{\nu} D_A - t_{\mu} t^{\nu} D_B] \omega_{\nu}.\end{aligned}\quad (53)$$

A number of pieces of notation require explanation. The differential operators D_A and D_B are

$$D_A \equiv \Omega \left[\partial^2 + \frac{\Omega''}{\Omega} \right] \Omega, \quad (54)$$

$$D_B \equiv \Omega \left[\partial^2 + \frac{(\Omega^{-1})''}{\Omega^{-1}} \right] \Omega, \quad (55)$$

where $\partial^2 \equiv \eta^{\mu\nu} \partial_{\mu} \partial_{\nu}$ is the d'Alembertian in conformal coordinates. It is worth commenting that D_A is the kinetic operator for a massless, minimally coupled scalar. The kinetic operator for the (general) pseudograviton is

$$\begin{aligned}D_{\mu\nu}{}^{\rho\sigma} \equiv & \left[\frac{1}{2} \bar{\delta}_{\mu}^{(\rho} \bar{\delta}_{\nu}^{\sigma)} - \frac{1}{4} \eta_{\mu\nu} \eta^{\rho\sigma} - \frac{1}{2} t_{\mu} t_{\nu} t^{\rho} t^{\sigma} \right] D_A \\ & - t_{(\mu} \bar{\delta}_{\nu)}^{(\rho} t^{\sigma)} D_B + t_{\mu} t_{\nu} t^{\rho} t^{\sigma} D_B.\end{aligned}\quad (56)$$

Parenthesized indices are symmetrized, the symbol t_{μ} denotes

$$t_{\mu} \equiv \eta_{\mu 0}, \quad t^{\mu} = \delta_0^{\mu}, \quad (57)$$

and a bar above a Lorentz metric or a Kronecker delta symbol means that the zero component is projected out:

$$\bar{\eta}_{\mu\nu} \equiv \eta_{\mu\nu} + t_{\mu} t_{\nu}, \quad \bar{\delta}_{\mu}^{\nu} \equiv \delta_{\mu}^{\nu} + t_{\mu} t^{\nu}. \quad (58)$$

The quadratic Lagrangian involves two sorts of mixing: that between the spatial trace and ψ_{00} and that between ψ_{00} and the scalar ϕ . The first can be removed by the following simple field redefinition:

$$\psi_{ij} \equiv \zeta_{ij} + \delta_{ij} \zeta_{00}, \quad \psi_{0i} \equiv \zeta_{0i}, \quad \psi_{00} \equiv \zeta_{00}, \quad (59)$$

where small Latin letters denote spatial indices. In these variables the quadratic part of the Lagrangian becomes

$$\begin{aligned}\mathcal{L}_{\text{BRS}}^{(2)} = & \frac{1}{2} \zeta_{00} D_B \zeta_{00} - \kappa \Omega^2 \varphi_0'' \zeta_{00} \phi \\ & + \frac{1}{2} \phi \Omega \left[\partial^2 + \frac{1}{4} \kappa^2 \varphi_0'^2 + \frac{\varphi_0'''}{\varphi_0'} \right] \Omega \phi \\ & + \frac{1}{2} \zeta_{ij} \left(\frac{1}{2} \delta_{i(k} \delta_{l)j} - \frac{1}{4} \delta_{ij} \delta_{kl} \right) D_A \zeta_{kl} \\ & - \frac{1}{2} \zeta_{0i} \delta_{ij} D_B \zeta_{0j} + \bar{\omega}^{\mu} [\bar{\delta}_{\mu}^{\nu} D_A - t_{\mu} t^{\nu} D_B] \omega_{\nu}.\end{aligned}\quad (60)$$

For a general $a_0(t)$ there is no local change of variables which removes the mixing between ζ_{00} and ϕ off shell. However, it is easy to diagonalize the linearized *field equations* which determine the on shell mode solutions. Canonical quantization of the linearized theory can then be invoked to expand the original quantum fields in terms of creation and annihilation operators. It is straightforward to use these expansions to express the propagators as mode sums. At the cost of some mixed propagators we will eventually obtain a complete expression of the Feynman rules in terms of the field variables $\psi_{\mu\nu}$, $\bar{\omega}^{\mu}$, ω_{ν} , and ϕ .

It is simplest to absorb a factor of Ω into ζ_{00} and ϕ :

$$z \equiv \Omega \zeta_{00}, \quad f \equiv \Omega \phi. \quad (61)$$

The linearized equations for the z - f system are

$$\left(\partial^2 + \frac{1}{4}\kappa^2\varphi_0'^2\right)z - \kappa\varphi_0''f = 0, \quad (62)$$

$$-\kappa\varphi_0''z + \left(\partial^2 + \frac{1}{4}\kappa^2\varphi_0'^2 + \frac{\varphi_0'''}{\varphi_0'}\right)f = 0. \quad (63)$$

Differentiating Eq. (62) and adding it to $\frac{1}{2}\kappa\varphi_0'$ times Eq. (63) gives the first of our diagonalized field equations

$$\left(\partial^2 + \frac{1}{4}\kappa^2\varphi_0'^2\right)\left[z' + \frac{1}{2}\kappa\varphi_0'f\right] = 0. \quad (64)$$

The second diagonalized field equation comes from differentiating Eq. (63) and adding it to $\frac{1}{2}\kappa\varphi_0'$ times Eq. (62) minus φ_0''/φ_0' times Eq. (63):

$$\left(\partial^2 + \frac{1}{4}\kappa^2\varphi_0'^2 - \frac{\varphi_0'''}{\varphi_0'} + 2\frac{\varphi_0''^2}{\varphi_0'^2}\right)\left[\frac{1}{2}\kappa\varphi_0'z + f' - \frac{\varphi_0''}{\varphi_0'}f\right] = 0. \quad (65)$$

The preceding discussion implies that the diagonal variables are

$$x(\eta, \vec{x}) \equiv z'(\eta, \vec{x}) + \frac{1}{2}\kappa\varphi_0'(\eta)f(\eta, \vec{x}), \quad (66)$$

$$y(\eta, \vec{x}) \equiv \frac{1}{2}\kappa\varphi_0'(\eta)z(\eta, \vec{x}) + f'(\eta, \vec{x}) - \frac{\varphi_0''(\eta)}{\varphi_0'(\eta)}f(\eta, \vec{x}). \quad (67)$$

Since conformal time derivatives appear in the transformation its inverse cannot be local in time for the off shell fields. However, by using the linearized field equations one can obtain the following expressions for the conformal time derivatives of x and y :

$$x' = \left(\nabla^2 + \frac{1}{4}\kappa^2\varphi_0'^2\right)z + \frac{1}{2}\kappa\varphi_0'f' - \frac{1}{2}\kappa\varphi_0''f, \quad (68)$$

$$y' = \frac{1}{2}\kappa\varphi_0'z' - \frac{1}{2}\kappa\varphi_0''z - \frac{\varphi_0''}{\varphi_0'}f' + \left(\nabla^2 + \frac{1}{4}\kappa^2\varphi_0'^2 + \frac{\varphi_0''^2}{\varphi_0'^2}\right)f. \quad (69)$$

Eliminating z' and f' gives the following on shell inverse transformation:

$$z = \frac{1}{\nabla^2}\left[x' - \frac{1}{2}\kappa\varphi_0'y\right], \quad (70)$$

$$f = \frac{1}{\nabla^2}\left[-\frac{1}{2}\kappa\varphi_0'x + y' + \frac{\varphi_0''}{\varphi_0'}y\right]. \quad (71)$$

We stress that since the linearized field equations have been used these relations apply only to the on shell mode solutions, not to the off shell fields.

The mode equations for all the fields—including $\Omega\zeta_{ij}$, $\Omega\zeta_{0i}$, $\Omega\bar{\omega}^\mu$, and $\Omega\omega_\nu$ —can be given a simple, unified treatment. There are three types of modes which we shall call A, B, and C. They are defined as the plane wave solutions annihilated by the following differential operator:

$$\mathcal{D}_I \equiv \partial^2 + \frac{\theta_I''}{\theta_I} \rightarrow -\left\{\frac{d^2}{d\eta^2} + k^2 - \frac{\theta_I''(\eta)}{\theta_I(\eta)}\right\}, \quad (72)$$

where $k \equiv \|\vec{k}\|$ and the various $\theta_I(\eta)$'s are

$$\theta_A \equiv \Omega, \quad \theta_B \equiv \Omega^{-1}, \quad \theta_C \equiv -\frac{2}{\kappa} \frac{\Omega'}{\Omega^2\varphi_0'} = a_0^{-1} \frac{H}{\sqrt{-\dot{H}}}. \quad (73)$$

Briefly, the spatial polarizations $\Omega\zeta_{ij}$, $\Omega\bar{\omega}^\mu$, and $\Omega\omega_j$ are comprised of A modes, the mixed polarizations $\Omega\zeta_{0i}$, $\Omega\bar{\omega}^0$, and $\Omega\omega_0$ are made of B modes, as is the diagonal variable x , and the other diagonal variable y consists of C modes. Because quantization was accomplished by adding a gauge fixing term most of the linearized fields harbor unphysical quanta. Physical gravitons are A modes that reside in ζ_{ij} ; the physical scalar is a C mode in y .

We will return in Sec. VI to the problem of obtaining useful approximations for the mode functions but we proceed, for now, as though they are known. We define $Q_I(\eta, k)$ as descending by perturbative iteration (explained in Sec. VI) from the pure negative frequency solution for wave number $k \equiv \|\vec{k}\|$. We also assume it has been canonically normalized:

$$Q_I(\eta, k)Q_I^{*'}(\eta, k) - Q_I'(\eta, k)Q_I^*(\eta, k) = i. \quad (74)$$

From canonically quantizing the quadratic action (60) one finds that the fields $z(\eta, \vec{x})$ and $z'(\eta, \vec{x})$ form a conjugate pair. The same is true for $f(\eta, \vec{x})$ and $f'(\eta, \vec{x})$, so the only nonzero equal-time commutators involving these fields are

$$[z(\eta, \vec{x}), z'(\eta, \vec{y})] = i\delta^3(\vec{x} - \vec{y}) = [f(\eta, \vec{x}), f'(\eta, \vec{y})]. \quad (75)$$

From their definitions (66), (67) and the on shell relations (68), (69) one can easily check that the only nonzero equal-time commutators in the x - y sector are

$$[x(\eta, \vec{x}), x'(\eta, \vec{y})] = -i\nabla^2\delta^3(\vec{x} - \vec{y}) = [y(\eta, \vec{x}), y'(\eta, \vec{y})]. \quad (76)$$

We can realize these commutation relations with conventionally normalized creation and annihilation operators ($X, X^\dagger$) and ($Y, Y^\dagger$):

$$[X(\vec{k}), X^\dagger(\vec{p})] = (2\pi)^3\delta^3(\vec{k} - \vec{p}) = [Y(\vec{k}), Y^\dagger(\vec{p})]. \quad (77)$$

Since $\mathcal{D}_B x(\eta, \vec{x}) = 0$ we expand x using B modes,

$$x(\eta, \vec{x}) = \int \frac{d^3k}{(2\pi)^3} k \{X(\vec{k})Q_B(\eta, k)e^{i\vec{k}\cdot\vec{x}} + X^\dagger(\vec{k})Q_B^*(\eta, k)e^{i\vec{k}\cdot\vec{x}}\}. \quad (78)$$

Since $\mathcal{D}_C y(\eta, \vec{x}) = 0$ we expand y using C modes,

$$y(\eta, \vec{x}) = \int \frac{d^3 k}{(2\pi)^3} k \{ Y(\vec{k}) Q_C(\eta, k) e^{i\vec{k} \cdot \vec{x}} + Y^\dagger(\vec{k}) Q_C^*(\eta, k) e^{i\vec{k} \cdot \vec{x}} \}. \quad (79)$$

The on shell transformations (70),(71) allow us finally to give operator expansions for z and f :

$$z(\eta, \vec{x}) = \int \frac{d^3 k}{(2\pi)^3} \frac{e^{1 \cdot \vec{x}}}{k} \left\{ -X(\vec{k}) Q_B'(\eta, k) e^{i\vec{k} \cdot \vec{x}} + Y(\vec{k}) \frac{1}{2} \kappa \varphi_0'(\eta) Q_C(\eta, k) e^{i\vec{k} \cdot \vec{x}} + \text{c.c.} \right\}, \quad (80)$$

$$f(\eta, \vec{x}) = \int \frac{d^3 k}{(2\pi)^3} \frac{e^{1 \cdot \vec{x}}}{k} \left\{ X(\vec{k}) \frac{1}{2} \kappa \varphi_0'(\eta) Q_B(\eta, k) e^{i\vec{k} \cdot \vec{x}} - Y(\vec{k}) \left[\frac{d}{d\eta} + \frac{\varphi_0''(\eta)}{\varphi_0'(\eta)} \right] Q_C(\eta, k) e^{i\vec{k} \cdot \vec{x}} + \text{c.c.} \right\}. \quad (81)$$

The rest is a standard exercise in free field theory. We choose the state $|0\rangle$ to obey

$$X(\vec{k})|0\rangle = 0 = Y(\vec{k})|0\rangle. \quad (82)$$

The various propagators can be most conveniently expressed in terms of the mode sum⁷ $i\delta_I(x; x')$

$$\begin{aligned} i\delta_I(x; x') &= -\frac{1}{\nabla^2} \int \frac{d^3 k}{(2\pi)^3} e^{i\vec{k} \cdot \Delta \vec{x} - \epsilon k} \\ &\times \{ \theta(\Delta \eta) Q_I(\eta, k) Q_I^*(\eta', k) \\ &+ \theta(-\Delta \eta) Q_I^*(\eta, k) Q_I(\eta', k) \}, \quad (83) \\ &= \frac{1}{2\pi^2} \int_0^\infty dk \frac{\sin(k\Delta x)}{k\Delta x} \\ &\times e^{-\epsilon k} \{ \theta(\Delta \eta) Q_I(\eta, k) Q_I^*(\eta', k) \\ &+ \theta(-\Delta \eta) Q_I^*(\eta, k) Q_I(\eta', k) \}, \quad (84) \end{aligned}$$

where we define the following conformal coordinate differences:

$$\Delta \eta \equiv \eta - \eta', \quad \Delta \vec{x} \equiv \vec{x} - \vec{x}', \quad \Delta x \equiv \|\vec{x} - \vec{x}'\|. \quad (85)$$

Expanding the various time-ordered products and exploiting Eq. (82) leads to the following expressions for the propagators:

⁷Note that we have introduced a time-independent, ultraviolet convergence factor of $e^{-\epsilon k}$. This corresponds to an exponential mode cutoff on the initial value surface.

$$\begin{aligned} \langle 0 | T \{ z(\eta, \vec{x}) z(\eta', \vec{x}') \} | 0 \rangle &= \frac{\partial}{\partial \eta} \frac{\partial}{\partial \eta'} i\delta_B(x; x') \\ &+ \frac{i}{\nabla^2} \delta^4(x - x') \\ &+ \frac{1}{4} \kappa^2 \varphi_0'(\eta) \varphi_0'(\eta') i\delta_C(x; x'), \quad (86) \end{aligned}$$

$$\begin{aligned} \langle 0 | T \{ z(\eta, \vec{x}) f(\eta', \vec{x}') \} | 0 \rangle &= -\frac{\partial}{\partial \eta} \frac{1}{2} \kappa \varphi_0'(\eta') i\delta_B(x; x') \\ &- \frac{1}{2} \kappa \varphi_0'(\eta) \left[\frac{\partial}{\partial \eta'} + \frac{\varphi_0''}{\varphi_0'} \right] \\ &\times i\delta_C(x; x'), \quad (87) \end{aligned}$$

$$\begin{aligned} \langle 0 | T \{ f(\eta, \vec{x}) f(\eta', \vec{x}') \} | 0 \rangle &= \frac{1}{4} \kappa^2 \varphi_0'(\eta) \varphi_0'(\eta') i\delta_B(x; x') \\ &+ \left[\frac{\partial}{\partial \eta} + \frac{\varphi_0''}{\varphi_0'} \right] \left[\frac{\partial}{\partial \eta'} + \frac{\varphi_0''}{\varphi_0'} \right] \\ &\times i\delta_C(x; x') + \frac{i}{\nabla^2} \delta^4(x - x'). \quad (88) \end{aligned}$$

Now we restore the factors of Ω^{-1} . These can be used to convert the conformal time derivatives to comoving time

$$\Omega^{-1}(\eta) \frac{\partial}{\partial \eta} = \frac{\partial}{\partial t}, \quad \Omega^{-1} \varphi_0' = \dot{\varphi}_0. \quad (89)$$

The three z - f propagators become

$$\begin{aligned} i\Delta_\alpha(x; x') &\equiv \langle 0 | T \{ \psi_{00}(\eta, \vec{x}) \psi_{00}(\eta', \vec{x}') \} | 0 \rangle \\ &= \frac{\partial}{\partial t} \frac{\partial}{\partial t'} i\delta_B(x; x') + \frac{1}{4} \kappa^2 \dot{\varphi}_0(t) \dot{\varphi}_0(t') i\delta_C(x; x') \\ &+ \frac{i\Omega^{-1}\Omega'^{-1}}{\nabla^2} \delta^4(x - x'), \quad (90) \end{aligned}$$

$$\begin{aligned} i\Delta_\beta(x; x') &\equiv \langle 0 | T \{ \psi_{00}(\eta, \vec{x}) \phi(\eta', \vec{x}') \} | 0 \rangle \\ &= -\frac{\partial}{\partial t} \frac{1}{2} \kappa \dot{\varphi}_0(t') i\delta_B(x; x') - \frac{1}{2} \kappa \dot{\varphi}_0(t) \\ &\times \left[\frac{\partial}{\partial t'} + H + \frac{\ddot{\varphi}_0}{\dot{\varphi}_0} \right] i\delta_C(x; x'), \quad (91) \end{aligned}$$

$$\begin{aligned}
i\Delta_\gamma(x;x') &\equiv \langle 0 | T \{ \phi(\eta, \vec{x}) \phi(\eta', \vec{x}') \} | 0 \rangle \\
&= \frac{1}{4} \kappa^2 \dot{\varphi}_0(t) \dot{\varphi}_0(t') i \delta_B(x;x') + \left[\frac{\partial}{\partial t} + H + \frac{\ddot{\varphi}_0}{\dot{\varphi}_0} \right] \\
&\quad \times \left[\frac{\partial}{\partial t'} + H + \frac{\ddot{\varphi}_0}{\dot{\varphi}_0} \right] i \delta_C(x;x') \\
&\quad + \frac{i\Omega^{-1}\Omega'^{-1}}{\nabla^2} \delta^4(x-x'). \quad (92)
\end{aligned}$$

The unmixed propagators can also be represented as mode sums

$$i\Delta_I(x;x) \equiv \Omega^{-1}(\eta) \Omega^{-1}(\eta') (-\nabla^2) i\delta_I(x;x). \quad (93)$$

The (general) pseudograviton propagator is

$$\begin{aligned}
\langle 0 | T \{ \psi_{\mu\nu}(x) \psi_{\rho\sigma}(x') \} | 0 \rangle \\
&= i\Delta_A(x;x') 2[\bar{\eta}_{\mu(\rho} \bar{\eta}_{\sigma)\nu} - \bar{\eta}_{\mu\nu} \bar{\eta}_{\rho\sigma}] \\
&\quad - 4i\Delta_B(x;x') t_{(\mu} \bar{\eta}_{\nu)(\rho} t_{\sigma)} \\
&\quad + i\Delta_\alpha(x;x') [\bar{\eta}_{\mu\nu} + t_\mu t_\nu] [\bar{\eta}_{\rho\sigma} + t_\rho t_\sigma]. \quad (94)
\end{aligned}$$

The other propagators are

$$\langle 0 | T \{ \psi_{\mu\nu}(x) \phi(x') \} | 0 \rangle = i\Delta_\beta(x;x') [\bar{\eta}_{\mu\nu} + t_\mu t_\nu], \quad (95)$$

$$\langle 0 | T \{ \phi(x) \phi(x') \} | 0 \rangle = i\Delta_\gamma(x;x'), \quad (96)$$

$$\langle 0 | T \{ \omega_\mu(x) \bar{\omega}_\nu(x') \} | 0 \rangle = i\Delta_A(x;x') \bar{\eta}_{\mu\nu} - i\Delta_B(x;x') t_\mu t_\nu. \quad (97)$$

All interactions between (general) pseudogravitons and scalars can be obtained by expanding the invariant Lagrangian (50) using the following identities:

$$\tilde{g}^{\mu\nu} = \eta^{\mu\nu} - \kappa \psi^{\mu\nu} + \kappa^2 \psi^{\mu\rho} \psi_\rho^\nu - \dots, \quad (98)$$

$$\sqrt{-\tilde{g}} = 1 + \frac{1}{2} \kappa \psi + \kappa^2 \left(\frac{1}{8} \psi^2 - \frac{1}{4} \psi^{\rho\sigma} \psi_{\rho\sigma} \right) + \dots \quad (99)$$

Interactions involving $\bar{\omega}^\mu$ and ω_ν can be read from the ghost Lagrangian, which we have simplified by neglecting some total derivatives

$$\begin{aligned}
\mathcal{L}_{\text{ghost}} &= -\Omega \bar{\omega}^\mu \delta F_\mu, \quad (100) \\
&= \bar{\omega}^\mu [\bar{\delta}_\mu^\nu D_A - t_\mu t^\nu D_B] \omega_\nu + \kappa (\Omega^2 \bar{\omega}^\mu)_{,\mu} \\
&\quad \times \left\{ \psi^{\nu\rho} \omega_{\nu,\rho} + \frac{1}{2} \psi^\nu \omega_\nu - \frac{\Omega'}{\Omega} \psi \omega_0 \right\} - \kappa \Omega^2 \bar{\omega}^{\mu,\nu} \\
&\quad \times \left\{ \psi_{\mu\rho} \omega_{,\nu}^\rho + \psi_{\nu\rho} \omega_{,\mu}^\rho + \psi_{\mu\nu,\rho} \omega^\rho - 2 \frac{\Omega'}{\Omega} \psi_{\mu\nu} \omega_0 \right\} \\
&\quad + \kappa^2 \Omega^2 \varphi_0' \bar{\omega}_0 \phi_{,\nu} \omega^\nu. \quad (101)
\end{aligned}$$

V. ATTACHING EXTERNAL LINES

The purpose of this section is to explain how we pass from the amputated one-point functions which are actually computed to physical observables. We begin by expressing the effective Hubble constant in terms of the (nonamputated) one-point function. The rest of the section is devoted to the procedure for attaching the retarded propagators, needed in the Schwinger formalism [5], to convert amputated one-point functions into their nonamputated cognates. Although an exact solution is obtained we specialize it, at the very end of the section, to leading order in the slow roll approximation.

Both the initial state and the evolution equations are homogeneous and isotropic. It follows that the expectation values of the (general) pseudograviton field and the scalar can be expressed in terms of three functions of η . It will simplify some of the later formulas if we choose to think of these quantities as functions of the comoving time t of the perturbative background, even though we still are expressing them in conformal coordinates⁸

$$\langle 0 | \kappa \psi_{\mu\nu}(\eta, \vec{x}) | 0 \rangle = A(t) \bar{\eta}_{\mu\nu} + C(t) t_\mu t_\nu, \quad (102)$$

$$\langle 0 | \kappa \phi(\eta, \vec{x}) | 0 \rangle = D(t). \quad (103)$$

Note that we cannot assume $C = -A$ since the expectation value may not be conformal in the perturbative coordinate system. None of these quantities is itself physical but they can be combined to produce observables. We first construct the invariant element to infer the true scale factor and comoving time $\bar{t}$ of the expectation value of the metric

$$-d\bar{t}^2 + a^2(\bar{t}) d\vec{x} \cdot d\vec{x} = \Omega^2 \{ - (1-C) d\eta^2 + (1+A) d\vec{x} \cdot d\vec{x} \}, \quad (104)$$

$$\begin{aligned}
&= -[1-C(t)] dt^2 + a_0^2(t) \\
&\quad \times [1+A(t)] d\vec{x} \cdot d\vec{x}. \quad (105)
\end{aligned}$$

One physical observable is the effective Hubble constant expressed as a function of the comoving time $\bar{t}$:

$$H_{\text{eff}}(\bar{t}) \equiv \frac{d}{d\bar{t}} \ln[a(\bar{t})] = \frac{1}{\sqrt{1-C(t)}} \left\{ H(t) + \frac{1}{2} \frac{\dot{A}(t)}{1+A(t)} \right\}. \quad (106)$$

If the scalar can be measured then its expectation value is also an observable when expressed as a function of the comoving time $\bar{t}$. We shall call this variable $\Phi(\bar{t})$:

$$\Phi(\bar{t}) \equiv \varphi_0(t) + \frac{1}{\kappa} D(t). \quad (107)$$

⁸Recall that the relation between t and η is

$$dt = \Omega(\eta) d\eta \Leftrightarrow \frac{dt}{a_0(t)} = d\eta.$$

What we actually compute are not the expectation values of $\kappa\psi_{\mu\nu}$ and $\kappa\phi$ but rather the *amputated* expectation values with the external propagators removed. We will use Greek letters to denote the three functions of t which describe these amputated quantities

$$\begin{aligned} \alpha(t) \bar{\eta}_{\mu\nu} + \gamma(t) t_\mu t_\nu &\equiv D_{\mu\nu}{}^{\rho\sigma} \langle 0 | \kappa\psi_{\rho\sigma}(\eta, \vec{x}) | 0 \rangle \\ &\quad - \kappa\Omega^2 \varphi_0'' t_\mu t_\nu \langle 0 | \kappa\phi(\eta, \vec{x}) | 0 \rangle, \end{aligned} \quad (108)$$

$$\begin{aligned} \delta(t) &\equiv -\kappa\Omega^2 \varphi_0'' t^\rho t^\sigma \langle 0 | \kappa\psi_{\rho\sigma}(\eta, \vec{x}) | 0 \rangle \\ &\quad + \Omega \left(\partial^2 + \frac{1}{4} \kappa^2 \varphi_0'^2 + \frac{\varphi_0'''}{\varphi_0'} \right) \Omega \langle 0 | \kappa\phi(\eta, \vec{x}) | 0 \rangle. \end{aligned} \quad (109)$$

Contracting with the kinetic operator (56) and isolating distinct tensor components gives three relations

$$\alpha = -\frac{1}{4} D_A (A - C), \quad (110)$$

$$\gamma = \frac{3}{4} D_A (A - C) + D_B C - \kappa\Omega^2 \varphi_0'' D, \quad (111)$$

$$\delta = -\kappa\Omega^2 \varphi_0'' C + \Omega \left[-\frac{d^2}{d\eta^2} + \frac{1}{4} \kappa^2 \varphi_0''^2 + \frac{\varphi_0'''}{\varphi_0'} \right] \Omega D. \quad (112)$$

Since it is from A and C that H_{eff} is constructed, we must invert these relations.

We employ the Schwinger formalism [5] in order to get true expectation values rather than in-out matrix elements. An important feature of this formalism is that external legs are retarded propagators. This means that the coupled differential equations in Eqs. (110)–(112) must be inverted using retarded boundary conditions

$$0 = A(0) = C(0) = D(0), \quad (113)$$

$$0 = \dot{A}(0) = \dot{C}(0) = \dot{D}(0). \quad (114)$$

Now it happens that every differential equation we have to solve can be cast in the form

$$\mathcal{D}f(t) \equiv \left(-\frac{d^2}{d\eta^2} + \frac{\theta''}{\theta} \right) f(t) = g(t). \quad (115)$$

This is fortunate because the retarded solution can be simply expressed as a double integral

$$\begin{aligned} f(t) &= \mathcal{D}^{-1}(g) \equiv -\theta(\eta) \int_0^t dt_1 a_0^{-1}(t_1) \theta^{-2}(\eta_1) \\ &\quad \times \int_0^{t_1} dt_2 a_0^{-1}(t_2) \theta(\eta_2) g(t_2). \end{aligned} \quad (116)$$

It simplifies the algebra somewhat to multiply the amputated quantities by Ω^{-1} and their unamputated descendants by Ω . We denote the rescaled variables by a tilde:

$$\tilde{A} \equiv \Omega A, \quad \tilde{C} \equiv \Omega C, \quad \tilde{D} \equiv \Omega D, \quad (117)$$

$$\tilde{\alpha} \equiv \Omega^{-1} \alpha, \quad \tilde{\gamma} \equiv \Omega^{-1} \gamma, \quad \tilde{\delta} \equiv \Omega^{-1} \delta. \quad (118)$$

In this notation the equations we must invert are

$$\tilde{\alpha} = -\frac{1}{4} \mathcal{D}_A (\tilde{A} - \tilde{C}), \quad (119)$$

$$\tilde{\gamma} = \frac{3}{4} \mathcal{D}_A (\tilde{A} - \tilde{C}) + \mathcal{D}_B \tilde{C} - \kappa \varphi_0'' \tilde{D}, \quad (120)$$

$$\tilde{\delta} = -\kappa \varphi_0'' \tilde{C} + \left[-\frac{d^2}{d\eta^2} + \frac{1}{4} \kappa^2 \varphi_0''^2 + \frac{\varphi_0'''}{\varphi_0'} \right] \tilde{D}, \quad (121)$$

where $\mathcal{D}_A$ and $\mathcal{D}_B$ have the form (115) with $\theta_A = \Omega$ and $\theta_B = \Omega^{-1}$. Equation (119) implies

$$\tilde{A} = \tilde{C} + \mathcal{D}_A^{-1} (-4\tilde{\alpha}). \quad (122)$$

Substituting this into Eq. (120) gives

$$3\tilde{\alpha} + \tilde{\gamma} = \mathcal{D}_B \tilde{C} - \kappa \varphi_0'' \tilde{D}, \quad (123)$$

which, with Eq. (121), is similar to the coupled z - f system of Sec. IV. Paralleling the analysis of that section we differentiate Eq. (123) and add it to $\frac{1}{2} \kappa \varphi_0'$ times Eq. (121) to obtain

$$\mathcal{D}_B \left(\tilde{C}' + \frac{1}{2} \kappa \varphi_0' \tilde{D} \right) = 3\tilde{\alpha}' + \tilde{\gamma}' + \frac{1}{2} \kappa \varphi_0' \tilde{\delta}. \quad (124)$$

Differentiating Eq. (121) and adding it to $\frac{1}{2} \kappa \varphi_0'$ times Eq. (123) minus φ_0''/φ_0' times Eq. (121) gives

$$\mathcal{D}_C \left(\frac{1}{2} \kappa \varphi_0' \tilde{C} + \tilde{D}' - \frac{\varphi_0''}{\varphi_0'} \tilde{D} \right) = \frac{1}{2} \kappa \varphi_0' (3\tilde{\alpha} + \tilde{\gamma}) + \tilde{\delta}' - \frac{\varphi_0''}{\varphi_0'} \tilde{\delta}, \quad (125)$$

where $\mathcal{D}_C$ has the form (115) with $\theta_C = -2\kappa^{-1}\Omega^{-2}\Omega'/\varphi_0'$.

Of course we can invert the differential operators in the last two equations

$$\tilde{C}' + \frac{1}{2} \kappa \varphi_0' \tilde{D} = \mathcal{D}_B^{-1} \left(3\tilde{\alpha}' + \tilde{\gamma}' + \frac{1}{2} \kappa \varphi_0' \tilde{\delta} \right), \quad (126)$$

$$\frac{1}{2} \kappa \varphi_0' \tilde{C} + \tilde{D}' - \frac{\varphi_0''}{\varphi_0'} \tilde{D} = \mathcal{D}_C^{-1} \left(\frac{1}{2} \kappa \varphi_0' (3\tilde{\alpha} + \tilde{\gamma}) + \tilde{\delta}' - \frac{\varphi_0''}{\varphi_0'} \tilde{\delta} \right), \quad (127)$$

but this still leaves derivatives on $\tilde{C}$ and $\tilde{D}$. These derivatives cannot be removed as we did in Sec. IV because the Laplacian vanishes for spatially homogeneous functions. What we must do instead is to divide Eq. (123) by φ_0' and add it to $2\varphi_0''/\varphi_0'^2$ times Eq. (126):

$$\mathcal{D}_C \left(\frac{\tilde{C}}{\varphi_0'} \right) = \frac{3\tilde{\alpha} + \tilde{\gamma}}{\varphi_0'} + 2 \frac{\varphi_0''}{\varphi_0'^2} \mathcal{D}_B^{-1} \left(3\tilde{\alpha}' + \tilde{\gamma}' + \frac{1}{2} \kappa \varphi_0' \tilde{\delta} \right). \quad (128)$$

Dividing Eq. (121) by φ_0' and adding it to $2\varphi_0''/\varphi_0'^2$ times Eq. (127) gives a similar relation for $\tilde{D}$:

$$\mathcal{D}_B\left(\frac{\tilde{D}}{\varphi'_0}\right) = \frac{\tilde{\delta}}{\varphi'_0} + 2\frac{\varphi''_0}{\varphi'^2_0}\mathcal{D}_C^{-1}\left(\frac{1}{2}\kappa\varphi'_0(3\tilde{\alpha}+\tilde{\gamma})+\tilde{\delta}'-\frac{\varphi''_0}{\varphi'_0}\tilde{\delta}\right). \quad (129)$$

Putting everything together gives the following solution for the unamputated coefficient functions:

$$A = \Omega^{-1}\mathcal{D}_A^{-1}(-4\tilde{\alpha}) + \frac{\varphi'_0}{\Omega}\mathcal{D}_C^{-1} \times \left\{ \frac{3\tilde{\alpha}+\tilde{\gamma}}{\varphi'_0} + 2\frac{\varphi''_0}{\varphi'^2_0}\mathcal{D}_B^{-1}\left(3\tilde{\alpha}'+\tilde{\gamma}'+\frac{1}{2}\kappa\varphi'_0\tilde{\delta}\right) \right\}, \quad (130)$$

$$C = \frac{\varphi'_0}{\Omega}\mathcal{D}_C^{-1}\left\{ \frac{3\tilde{\alpha}+\tilde{\gamma}}{\varphi'_0} + 2\frac{\varphi''_0}{\varphi'^2_0}\mathcal{D}_B^{-1} \times \left(3\tilde{\alpha}'+\tilde{\gamma}'+\frac{1}{2}\kappa\varphi'_0\tilde{\delta}\right) \right\}, \quad (131)$$

$$D = \frac{\varphi'_0}{\Omega}\mathcal{D}_B^{-1}\left\{ \frac{\tilde{\delta}}{\varphi'_0} + 2\frac{\varphi''_0}{\varphi'^2_0}\mathcal{D}_C^{-1} \times \left(\frac{1}{2}\kappa\varphi'_0(3\tilde{\alpha}+\tilde{\gamma})+\tilde{\delta}'-\frac{\varphi''_0}{\varphi'_0}\tilde{\delta}\right) \right\}. \quad (132)$$

Recall that $\tilde{\alpha} \equiv \Omega^{-1}\alpha$, $\tilde{\gamma} \equiv \Omega^{-1}\gamma$ and $\tilde{\delta} \equiv \Omega^{-1}\delta$. The various inverse differential operators are defined by Eq. (116) with the following assignments for $\theta(\eta)$:

$$\theta_A = \Omega, \quad \theta_B = \Omega^{-1}, \quad \theta_C = -\frac{2}{\kappa}\frac{\Omega'}{\Omega^2\varphi'_0}. \quad (133)$$

For the potential $V(\varphi) = \frac{1}{2}m^2\varphi^2$ it happens that the leading order results for the amputated one-point functions consist of sums of terms with the general form

$$\alpha(t) = \alpha_N H^N(t) a_0^4(t) + \dots, \quad (134)$$

$$\gamma(t) = \gamma_N H^N(t) a_0^4(t) + \dots, \quad (135)$$

$$\delta(t) = \delta_N H^N(t) \left(\frac{\sqrt{-\dot{H}}}{H} \right) a_0^4(t) + \dots \quad (136)$$

The coefficients α_N , γ_N , and δ_N are constants. When the slow roll expansions of Sec. II are applied to the various integrations and differentiations in our formulas for A , C , and D , the following leading order results emerge:

$$A_N(t) = \frac{4\alpha_N}{3N} \left(\frac{H_I^N - H^N(t)}{H^2(t)} \right) \left(\frac{H^2}{-\dot{H}} \right) + \dots, \quad (137)$$

$$C_N(t) = \left[\left(\frac{3\alpha_N + \gamma_N}{2} \right) - \left(\frac{3\alpha_N + \gamma_N + \delta_N}{3N} \right) \right] \left(\frac{H_I^N - H^N(t)}{H^2(t)} \right) + \dots, \quad (138)$$

$$D_N(t) = - \left(\frac{3\alpha_N + \gamma_N + \delta_N}{3N} \right) \left(\frac{H_I^N - H^N(t)}{H^2(t)} \right) \left(\frac{H}{\sqrt{-\dot{H}}} \right) + \dots. \quad (139)$$

Here $H_I \equiv H(0)$ is the Hubble constant at the beginning of inflation. These results imply the following leading order shift in comoving time:

$$(\bar{t}-t)_N = -\frac{1}{2} \left[\frac{3}{2} \alpha_N - \left(\frac{3\alpha_N + \gamma_N + \delta_N}{3N} \right) \right] \times \left(\frac{H_I^N}{H^2} - \frac{N}{N-1} \frac{H_I^{N-1}}{H} + \frac{H^{N-2}}{N-1} \right) \left(\frac{H}{\sqrt{-\dot{H}}} \right) + \dots. \quad (140)$$

To leading order the proportional shift in the two observables is

$$\left(\frac{H_{\text{eff}} - H}{H} \right)_N = \frac{2}{3} \alpha_N H^{N-2} + \left[\left(\frac{3\alpha_N + \gamma_N}{4} \right) \left(\frac{N}{N-1} \right) - \left(\frac{3\alpha_N + \gamma_N + \delta_N}{6(N-1)} \right) \right] \left(\frac{H_I^{N-1} - H^{N-1}}{H} \right) + \dots, \quad (141)$$

$$\left(\frac{\Phi - \varphi_0}{\varphi_0} \right)_N = - \left(\frac{3\alpha_N + \gamma_N}{4} \right) \left[\frac{H_I^N}{H^2} - \frac{N}{N-1} \frac{H_I^{N-1}}{H} + \frac{H^{N-2}}{N-1} \right] - \left(\frac{3\alpha_N + \gamma_N + \delta_N}{6(N-1)} \right) [H^{-1}H_I^{N-1} - H^{N-2}] + \dots. \quad (142)$$

To obtain the full shift one sums the contributions for various different values of N .

VI. INFRARED PARTS OF PROPAGATORS

This section deals with a very important omission in Sec. IV. Although we were able there to express the various propagators as mode sums for an arbitrary scalar potential, we do not possess the corresponding mode *functions* $Q_I(\eta, k)$ for a general potential. This is a standard problem in the theory of cosmological perturbations [16,18] and we solve it in the standard way: by developing series solutions for the ultraviolet (early time) and infrared (late time) regimes. The normalization for the ultraviolet expansion derives from the flat space limit. We normalize the infrared expansion by matching its leading term with that of the ultraviolet expansion at the time when the physical wavelength

of each mode is just redshifting beyond the Hubble radius. (This is the chief approximation of the paper.) One then defines the “infrared part” of each propagator as that obtained from the leading order term of the infrared expansion. We report explicit results to leading order in the slow roll approximation.

Recall from Sec. IV that we have three kinds of plane wave mode solutions $Q_I(\eta, k)$. They obey the equation

$$\mathcal{D}_I Q_I(\eta, k) \equiv - \left\{ \frac{d}{d\eta^2} + k^2 - \frac{\theta_I''(\eta)}{\theta_I(\eta)} \right\} Q_I(\eta, k) = 0, \quad (143)$$

where the $\theta_I(\eta)$ ’s are

$$\theta_A \equiv \Omega, \quad \theta_B \equiv \Omega^{-1}, \quad \theta_C \equiv -\frac{2}{\kappa} \frac{\Omega'}{\Omega^2 \varphi_0'} = a_0^{-1} \frac{H}{\sqrt{-\dot{H}}}. \quad (144)$$

Of course Eq. (143) does not completely define the modes because there are two linearly independent solutions. We define $Q_I(\eta, k)$ as the solution of Eq. (143) which is canonically normalized

$$Q(\eta, k) Q^{*'}(\eta, k) - Q'(\eta, k) Q^*(\eta, k) = i \quad (145)$$

and descends by perturbative iteration from the negative frequency solution of the far ultraviolet.

The far ultraviolet is defined by $k^2 \gg \theta''/\theta$. At fixed k this condition will also be realized, in all models of inflation, as the conformal time approaches negative infinity. In the ultraviolet regime we build up normalized solutions by iterating the following equation:

$$Q_I(\eta, k) = \frac{1}{\sqrt{2k}} e^{-ik\eta} + \int_{-\infty}^{\eta} d\bar{\eta} \frac{1}{k} \sin[k(\eta - \bar{\eta})] \frac{\theta_I''(\bar{\eta})}{\theta_I(\bar{\eta})} Q_I(\bar{\eta}, k). \quad (146)$$

The result is a series in inverse powers of k . These solutions are obviously negative frequency in the far ultraviolet. Their Wronskian (145) is constant as a simple consequence of the mode equation (143) while its actual value derives from the fact that θ''/θ vanishes as the conformal time approaches negative infinity.

The far infrared is defined by $k^2 \ll \theta''/\theta$. At fixed k this condition will also be realized, in all models of inflation, as the conformal time approaches zero from below. One can find explicit solutions in the limit that the k^2 term is neglected. The first one has the same form for $I=A, B, C$:

$$Q_{10,I}(\eta) \equiv \theta_I(\eta). \quad (147)$$

The second is an integral whose convergence (for models of inflation) requires different limits for $I=A$,

$$Q_{20,A}(\eta) \equiv -\theta_I(\eta) \int_{\eta}^0 \frac{d\bar{\eta}}{\theta_A^2(\bar{\eta})} \quad (148)$$

and $I=B, C$,

$$Q_{20,I}(\eta) \equiv \theta_I(\eta) \int_{-\infty}^{\eta} \frac{d\bar{\eta}}{\theta_I^2(\bar{\eta})} \quad (I=B, C). \quad (149)$$

When k^2 is small but not zero one can build up solutions which descend from $Q_{i0,I}(\eta)$ by iterating with the appropriate Green’s function

$$Q_{i,I}(\eta, k) = Q_{i0,I}(\eta) - k^2 \int_{-\infty}^0 d\bar{\eta} \times G_{\text{app}}(\eta, \bar{\eta}) Q_{i,I}(\eta, k) \quad (i=1,2). \quad (150)$$

This obviously gives a series of increasing powers of k^2 . Here the “appropriate” Green’s function is chosen to make the integral converge. The four possibilities are

$$G_{\text{adv}}(\eta, \bar{\eta}) = +\theta(\bar{\eta} - \eta) Q_{10}(\eta) Q_{20}(\bar{\eta}) - \theta(\bar{\eta} - \eta) Q_{20}(\eta) Q_{10}(\bar{\eta}), \quad (151)$$

$$G_{12}(\eta, \bar{\eta}) = -\theta(\eta - \bar{\eta}) Q_{10}(\eta) Q_{20}(\bar{\eta}) - \theta(\bar{\eta} - \eta) Q_{20}(\eta) Q_{10}(\bar{\eta}), \quad (152)$$

$$G_{21}(\eta, \bar{\eta}) = +\theta(\eta - \bar{\eta}) Q_{20}(\eta) Q_{10}(\bar{\eta}) + \theta(\bar{\eta} - \eta) Q_{10}(\eta) Q_{20}(\bar{\eta}), \quad (153)$$

$$G_{\text{ret}}(\eta, \bar{\eta}) = +\theta(\eta - \bar{\eta}) Q_{20}(\eta) Q_{10}(\bar{\eta}) - \theta(\eta - \bar{\eta}) Q_{10}(\eta) Q_{20}(\bar{\eta}), \quad (154)$$

and it should be noted that one may have to switch from one to another midway through the iteration process.

Since the full infrared solutions $Q_{1,I}(\eta, k)$ and $Q_{2,I}(\eta, k)$ span the space of solutions to $\mathcal{D}_I = 0$, it must be possible to express the ultraviolet solutions as linear combinations:

$$Q_I(\eta, k) = q_1 Q_{1,I}(\eta, k) + q_2 Q_{2,I}(\eta, k). \quad (155)$$

If we had the full solutions it would be straightforward to determine the combination coefficients

$$q_1 = \frac{Q_{2,I}'(\eta, k) Q_I(\eta, k) - Q_{2,I}(\eta, k) Q_I'(\eta, k)}{Q_{1,I}(\eta, k) Q_{2,I}'(\eta, k) - Q_{2,I}(\eta, k) Q_{1,I}'(\eta, k)}, \quad (156)$$

$$q_2 = \frac{-Q_{1,I}'(\eta, k) Q_I(\eta, k) + Q_{1,I}(\eta, k) Q_I'(\eta, k)}{Q_{1,I}(\eta, k) Q_{2,I}'(\eta, k) - Q_{2,I}(\eta, k) Q_{1,I}'(\eta, k)}, \quad (157)$$

where any conformal time η could be chosen.

For most backgrounds we do not possess the full solutions—either in the ultraviolet or the infrared. However, it happens that one of the zeroth order infrared solutions—

either $Q_{10,I}(\eta)$ or $Q_{20,I}(\eta)$ —dominates the other and all corrections as the conformal time approaches zero from below. The standard approximation [16,18] is to match this solution with the zeroth order ultraviolet solution at the horizon crossing time η_* , whose defining condition is

$$k = H_* \Omega(\eta_*). \quad (158)$$

Then the behavior of the modes in the far infrared can be approximated as follows:

$$Q_I(\eta, k) \rightarrow \frac{Q_{i0,I}(\eta)}{Q_{i0,I}(\eta_*)} \frac{e^{-ik\eta_*}}{\sqrt{2k}}, \quad (159)$$

where i is either 1 or 2, depending upon which of the zeroth order infrared solutions dominates for the I mode.

For $I=A$ it is $Q_{10,A}(\eta) = \Omega(\eta)$ that dominates at late times. We can therefore write

$$Q_A(\eta, k) \rightarrow \Omega(\eta) \frac{H_*}{k} \frac{e^{-ik\eta_*}}{\sqrt{2k}}. \quad (160)$$

For $I=B$ $Q_{20,B} = \Omega^{-1}$ becomes irrelevant at late times. The dominant solution is

$$\begin{aligned} Q_{20,B}(\eta) &= \Omega^{-1} \int_{-\infty}^{\eta} d\bar{\eta} \Omega^2(\bar{\eta}) \\ &= \frac{1}{a_0(t)} \int_{-\infty}^t dt' a_0(t') \approx \frac{1}{H(t)}. \end{aligned} \quad (161)$$

We can therefore approximate the B modes as follows:

$$Q_B(\eta, k) \rightarrow \frac{1}{H(t)} H_* \frac{e^{-ik\eta_*}}{\sqrt{2k}}. \quad (162)$$

The case of $I=C$ requires a more extensive analysis. The first solution is down by an inverse scale factor but enhanced by the inverse of a slow roll parameter

$$Q_{10,C} = a_0^{-1} \frac{H}{\sqrt{-\dot{H}}}. \quad (163)$$

The second solution has to be reexpressed several times before it can be recognized as the dominant one:

$$Q_{20,C} = a_0^{-1} \frac{H}{\sqrt{-\dot{H}}} \int_{-\infty}^t dt' a_0(t') \frac{d}{dt'} \left(\frac{1}{H(t')} \right), \quad (164)$$

$$= \frac{1}{\sqrt{-\dot{H}}} \frac{d}{dt} \left\{ \frac{1}{a_0(t)} \int_{-\infty}^t dt' a_0(t') \right\}, \quad (165)$$

$$\approx \frac{\sqrt{-\dot{H}}}{H^2}. \quad (166)$$

After any significant amount of inflation the inverse scale factor is much smaller than $-\dot{H}/H^2$, so we can approximate the C modes as

$$Q_C(\eta, k) \rightarrow \frac{\sqrt{-\dot{H}(t)}}{H^2(t)} \frac{H_*^2}{\sqrt{-\dot{H}_*}} \frac{e^{-ik\eta_*}}{\sqrt{2k}}. \quad (167)$$

Although the approximations (160),(162),(167) we have just made may seem grotesque they are intimately related to the physics of superadiabatic amplification [8]. It is this phenomenon's vast enhancement of the usual 0-point energy which causes one of the zeroth order infrared solutions to dominate at late times. These approximations therefore isolate precisely the leading late time infrared effect we wish to study. In fact this is all that *can* be reliably studied using quantum general relativity. The ultraviolet regime, which these approximations fail to capture, cannot in any case be described perturbatively by quantum general relativity.

What remains is to implement the infrared approximations (160),(162),(167) in the various propagator mode sums derived in Sec. IV. Since we are only computing the amputated one-point function to one loop order, these propagators are all coincident. They may, however, bear derivatives. Since space derivatives add factors of k , which are small in the infrared, we need only consider time derivatives. We shall therefore set $\Delta x = 0$ but keep the two times nonzero. With these conventions all the propagators can be described in the standard form

$$i\Delta(x; x') \rightarrow f(t)g(t') \int dk \frac{H_*^2}{2k} h(k), \quad (168)$$

where it should be noted that H_* and $\dot{H}_*$ are functions of the co-moving wave number k , determined by the horizon crossing condition (158).

From Eq. (160) and the mode sums (93),(84) which define it we see that the infrared part of $i\Delta_A(x; x')$ approaches a constant

$$i\Delta_A(x; x') \rightarrow 1(1) \frac{1}{2\pi^2} \int dk \frac{H_*^2}{2k} (1). \quad (169)$$

The behavior of the B mode (162) and the mode sums (93),(84) which define $i\Delta_B(x; x')$ show that its infrared part actually falls off:

$$i\Delta_B(x; x') \rightarrow \frac{1}{a_0(t)H(t)} \frac{1}{a_0(t')H(t')} \frac{1}{2\pi^2} \int dk \frac{H_*^2}{2k} k^2. \quad (170)$$

Since the momentum integral is dominated by the ultraviolet, rather than the infrared, we conclude that the infrared part of this propagator is zero. The ψ_{00} propagator involves B modes (162) and C modes (167). From its defining relations (90), (84) we determine its infrared part to be

$$i\Delta_{\alpha}(x;x') \rightarrow \frac{-\dot{H}(t)}{H^2(t)} \frac{-\dot{H}(t')}{H^2(t')} \frac{1}{2\pi^2} \int dk \frac{H_*^2}{2k} \frac{H_*^2}{-\dot{H}_*}. \quad (171)$$

The mixed propagator is defined by relations (91),(84). Again applying the infrared mode approximations (162),(167) gives

$$i\Delta_{\beta}(x;x') \rightarrow \frac{-\dot{H}(t)}{H^2(t)} \frac{\sqrt{-\dot{H}(t')}}{H(t')} \frac{1}{2\pi^2} \int dk \frac{H_*^2}{2k} \frac{H_*^2}{-\dot{H}_*}. \quad (172)$$

The same infrared limits, applied to its defining relations (92),(84), reduce the scalar propagator to

$$i\Delta_{\gamma}(x;x') \rightarrow \frac{\sqrt{-\dot{H}(t)}}{H(t)} \frac{\sqrt{-\dot{H}(t')}}{H(t')} \frac{1}{2\pi^2} \int dk \frac{H_*^2}{2k} \frac{H_*^2}{-\dot{H}_*}. \quad (173)$$

By itself it is the strongest but one must allow for the effect of factors and derivatives from the interaction vertex.

Since modes only become infrared after horizon crossing, the momentum integrations are cut off at $k=H(t)a_0(t)$. The integral can be evaluated by first changing variables from k to the horizon crossing time t_*

$$dk \approx H^2(t_*)a_0(t_*)dt_* \quad (174)$$

and then employing the slow roll expansions of Sec. II

$$\frac{1}{2\pi^2} \int dk \frac{H_*^2}{2k} \left(\frac{H_*^2}{-\dot{H}_*} \right) \approx \frac{1}{4\pi^2} \int_0^t dt_* \frac{H^5(t_*)}{-\dot{H}}, \quad (175)$$

$$\approx \frac{1}{24\pi^2} \left(\frac{H_I^6 - H^6(t)}{\dot{H}^2} \right). \quad (176)$$

Note that $\dot{H}$ is approximately constant for the quadratic potential considered by Mukhanov, Abramo, and Brandenberger [1].

VII. AMPUTATED ONE-POINT FUNCTIONS

The purpose of this section is first to obtain one loop results for the three amputated one-point functions (108),(109) defined in Sec. V. We then exploit the technology of Sec. V to compute the two observables $H_{\text{eff}}(\bar{t})$ (106) and $\Phi(\bar{t})$ (107). We begin by explaining how cubic interactions are used to compute the amputated one-point functions. Throughout this section we are making both the slow roll approximation and specializing to a quadratic potential.

At one loop order the amputated one-point functions consist basically of coincident propagators contracted into cubic interaction vertices. In addition to the usual factor i there is an i from the kinetic operator acting on the external propagator. There is also an extra factor of κ from the fact that we define the one-point functions (102),(103) as $\langle 0|\kappa\psi_{\mu\nu}|0\rangle$ and $\langle 0|\kappa\phi|0\rangle$.

As an example let us consider the interaction

TABLE I. Vertex factors contracted into $\psi_{\alpha_1\beta_1}\psi_{\alpha_2\beta_2}\psi_{\alpha_3\beta_3}$ with $\psi_{\alpha_1\beta_1}$ external.

No.	Vertex factor	No.	Vertex factor
1	$\frac{1}{2}\kappa H\Omega^3\eta^{\alpha_1\beta_1}\eta^{\alpha_2\beta_2}\partial_2^{(\alpha_3\beta_3)}t^{\beta_3}$	22	$\frac{1}{2}\kappa\Omega^2\eta^{\alpha_2(\alpha_3}\eta^{\beta_3)\beta_2}\partial_3^{(\alpha_1\beta_1)}\partial_3^{\beta_2}$
2	$\frac{1}{2}\kappa H\Omega^3\eta^{\alpha_2\beta_2}\eta^{\alpha_3\beta_3}\partial_3^{(\alpha_1\beta_1)}t^{\beta_1}$	23	$\frac{1}{2}\kappa\Omega^2\eta^{\alpha_3(\alpha_1}\eta^{\beta_1)\beta_3}\partial_3^{(\alpha_2\beta_2)}\partial_2^{\beta_3}$
3	$\frac{1}{2}\kappa H\Omega^3\eta^{\alpha_3\beta_3}\eta^{\alpha_1\beta_1}\partial_1^{(\alpha_2\beta_2)}t^{\beta_2}$	24	$\frac{1}{2}\kappa\Omega^2\partial_2^{(\alpha_1}\eta^{\beta_1)(\alpha_3}\partial_3^{\beta_3)}\eta^{\alpha_2\beta_2}$
4	$-\kappa H\Omega^3\eta^{\alpha_1(\alpha_2}\eta^{\beta_2)\beta_1}\partial_1^{(\alpha_3\beta_3)}t^{\beta_3}$	25	$\frac{1}{2}\kappa\Omega^2\partial_3^{(\alpha_2}\eta^{\beta_2)(\alpha_1}\partial_1^{\beta_1)}\eta^{\alpha_3\beta_3}$
5	$-\kappa H\Omega^3\eta^{\alpha_2(\alpha_3}\eta^{\beta_3)\beta_2}\partial_2^{(\alpha_1\beta_1)}t^{\beta_1}$	26	$\frac{1}{2}\kappa\Omega^2\partial_1^{(\alpha_3}\eta^{\beta_3)(\alpha_2}\partial_2^{\beta_2)}\eta^{\alpha_1\beta_1}$
6	$-\kappa H\Omega^3\eta^{\alpha_3(\alpha_1}\eta^{\beta_1)\beta_3}\partial_1^{(\alpha_2\beta_2)}t^{\beta_2}$	27	$\frac{1}{2}\kappa\Omega^2\partial_2^{(\alpha_1}\eta^{\beta_1)(\alpha_2}\partial_3^{\beta_3)}\eta^{\alpha_3\beta_3}$
7	$-\kappa H\Omega^3t^{(\alpha_3}\eta^{\beta_3)(\alpha_1}\partial_2^{\beta_1)}\eta^{\alpha_2\beta_2}$	28	$\frac{1}{2}\kappa\Omega^2\partial_3^{(\alpha_2}\eta^{\beta_2)(\alpha_3}\partial_1^{\beta_3)}\eta^{\alpha_1\beta_1}$
8	$-\kappa H\Omega^3t^{(\alpha_1}\eta^{\beta_1)(\alpha_2}\partial_3^{\beta_2)}\eta^{\alpha_3\beta_3}$	29	$\frac{1}{2}\kappa\Omega^2\partial_1^{(\alpha_3}\eta^{\beta_3)(\alpha_1}\partial_2^{\beta_1)}\eta^{\alpha_2\beta_2}$
9	$-\kappa H\Omega^3t^{(\alpha_2}\eta^{\beta_2)(\alpha_3}\partial_1^{\beta_3)}\eta^{\alpha_1\beta_1}$	30	$\frac{1}{8}\kappa\Omega^2\eta^{\alpha_1\beta_1}\eta^{\alpha_2\beta_2}\eta^{\alpha_3\beta_3}\partial_2\cdot\partial_3$
10	$\frac{1}{4}\kappa\Omega^2\eta^{\alpha_1\beta_1}\partial_3^{(\alpha_2}\eta^{\beta_2)(\alpha_3}\partial_2^{\beta_3)}$	31	$\frac{1}{4}\kappa\Omega^2\eta^{\alpha_1\beta_1}\eta^{\alpha_2\beta_2}\eta^{\alpha_3\beta_3}\partial_3\cdot\partial_1$
11	$\frac{1}{4}\kappa\Omega^2\eta^{\alpha_2\beta_2}\partial_1^{(\alpha_3}\eta^{\beta_3)(\alpha_1}\partial_3^{\beta_1)}$	32	$-\frac{1}{2}\kappa\Omega^2\eta^{\alpha_1(\alpha_2}\eta^{\beta_2)\beta_1}\eta^{\alpha_3\beta_3}\partial_2\cdot\partial_3$
12	$\frac{1}{4}\kappa\Omega^2\eta^{\alpha_3\beta_3}\partial_2^{(\alpha_1}\eta^{\beta_1)(\alpha_2}\partial_1^{\beta_2)}$	33	$-\frac{1}{2}\kappa\Omega^2\eta^{\alpha_2(\alpha_3}\eta^{\beta_3)\beta_2}\eta^{\alpha_1\beta_1}\partial_3\cdot\partial_1$
13	$-\kappa\Omega^2\partial_3^{(\alpha_1}\eta^{\beta_1)(\alpha_2}\eta^{\beta_2)(\alpha_3}\partial_2^{\beta_3)}$	34	$-\frac{1}{2}\kappa\Omega^2\eta^{\alpha_3(\alpha_1}\eta^{\beta_1)\beta_3}\eta^{\alpha_2\beta_2}\partial_1\cdot\partial_2$
14	$-\kappa\Omega^2\partial_1^{(\alpha_2}\eta^{\beta_2)(\alpha_3}\eta^{\beta_3)(\alpha_1}\partial_3^{\beta_1)}$	35	$-\frac{1}{4}\kappa\Omega^2\partial_2^{(\alpha_1}\partial_3^{\beta_1)}\eta^{\alpha_2\beta_2}\eta^{\alpha_3\beta_3}$
15	$-\kappa\Omega^2\partial_2^{(\alpha_3}\eta^{\beta_3)(\alpha_1}\eta^{\beta_1)(\alpha_2}\partial_1^{\beta_2)}$	36	$-\frac{1}{2}\kappa\Omega^2\partial_3^{(\alpha_2}\partial_1^{\beta_2)}\eta^{\alpha_3\beta_3}\eta^{\alpha_1\beta_1}$
16	$-\frac{1}{2}\kappa\Omega^2\partial_3^{(\alpha_2}\eta^{\beta_2)(\alpha_1}\eta^{\beta_1)(\alpha_3}\partial_2^{\beta_3)}$	37	$-\frac{1}{8}\kappa\Omega^2\eta^{\alpha_1\beta_1}\eta^{\alpha_2(\alpha_3}\eta^{\beta_3)\beta_2}\partial_2\cdot\partial_3$
17	$-\kappa\Omega^2\partial_1^{(\alpha_3}\eta^{\beta_3)(\alpha_2}\eta^{\beta_2)(\alpha_1}\partial_3^{\beta_1)}$	38	$-\frac{1}{4}\kappa\Omega^2\eta^{\alpha_2\beta_2}\eta^{\alpha_3(\alpha_1}\eta^{\beta_1)\beta_3}\partial_3\cdot\partial_1$
18	$-\frac{1}{4}\kappa\Omega^2\eta^{\alpha_1\beta_1}\eta^{\alpha_2\beta_2}\partial_2^{(\alpha_3}\partial_3^{\beta_3)}$	39	$\frac{1}{2}\kappa\Omega^2\eta^{\alpha_1(\alpha_2}\eta^{\beta_2)(\alpha_3}\eta^{\beta_3)(\beta_1}\partial_2\cdot\partial_3$
19	$-\frac{1}{4}\kappa\Omega^2\eta^{\alpha_2\beta_2}\eta^{\alpha_3\beta_3}\partial_3^{(\alpha_1}\partial_1^{\beta_1)}$	40	$\kappa\Omega^2\eta^{\alpha_1(\alpha_2}\eta^{\beta_2)(\alpha_3}\eta^{\beta_3)(\beta_1}\partial_3\cdot\partial_1$
20	$-\frac{1}{4}\kappa\Omega^2\eta^{\alpha_3\beta_3}\eta^{\alpha_1\beta_1}\partial_1^{(\alpha_2}\partial_2^{\beta_2)}$	41	$\frac{1}{4}\kappa\Omega^2\partial_2^{(\alpha_1}\partial_3^{\beta_1)}\eta^{\alpha_2(\alpha_3}\eta^{\beta_3)\beta_2}$
21	$\frac{1}{2}\kappa\Omega^2\eta^{\alpha_1(\alpha_2}\eta^{\beta_2)\beta_1}\partial_2^{(\alpha_3}\partial_3^{\beta_3)}$	42	$\frac{1}{2}\kappa\Omega^2\partial_3^{(\alpha_2}\partial_1^{\beta_2)}\eta^{\alpha_3(\alpha_1}\eta^{\beta_1)\beta_3}$

$-\frac{1}{4}\kappa m^2\Omega^4\phi^2\psi$, which is one of the many cubic terms descending from the scalar mass. The external line can attach to any of the three quantum fields: $\phi^2\psi_{\mu\nu}\eta^{\mu\nu}$. When it attaches to one of the two scalar fields $\delta(t)$ receives the following contribution:

$$\left\{ i\kappa\left(-\frac{i}{4}\kappa m^2\Omega^4\right)2\langle 0|T[\phi(x)\psi(x')]|0\rangle\right\}_{x'=x} \\ =\left\{ \frac{1}{2}\kappa^2m^2\Omega^4i\Delta_\beta(x;x')(\bar{\eta}_{\mu\nu}+t_\mu t_\nu)\eta^{\mu\nu}\right\}_{x'=x}, \quad (177)$$

$$=\{\kappa^2m^2\Omega^4i\Delta_\beta(x;x')\}_{x'=x}. \quad (178)$$

Most of the coincidence limit is ultraviolet nonsense which quantum general relativity cannot be trusted to treat correctly and which must in any case have been subtracted off in order for inflation to begin in the first place. The time-dependent, physically significant part comes from the superadiabatically amplified infrared modes. From Eq. (172) we see that the leading effect from these is

$$\kappa^2m^2\Omega^4i\Delta_\beta(x;x)\xrightarrow{\text{IR}}-3\kappa^2\dot{H}a_0^4(t) \\ \times\frac{(-\dot{H})^{3/2}}{H^3(t)}\frac{1}{24\pi^2}\left(\frac{H_I^6-H^6(t)}{\dot{H}^2}\right), \\ =\frac{\kappa^2}{8\pi^2}\left(\frac{H_I^6-H^6(t)}{H^2(t)}\right)\left(\frac{\sqrt{-\dot{H}}}{H(t)}\right)a_0^4(t). \quad (179)$$

In the notation used at the end of Sec. V there are contributions for $N=-2$ and $N=+4$ with coefficients of $\kappa^2/(8\pi^2)$ times $+H_I^6$ and -1 , respectively.

When the external leg attaches to the pseudo-graviton field the interaction makes contributions to $\alpha(t)$ and $\gamma(t)$. Because $\psi=-\psi_{00}+\psi_{ii}$ these have opposite signs. The contribution for $\alpha(t)$ is

$$\left\{ i\kappa\left(-\frac{i}{4}\kappa m^2\Omega^4\right)\langle 0|T[\phi(x)\phi(x')]|0\rangle\right\}_{x'=x} \\ =\frac{1}{4}\kappa^2m^2\Omega^4i\Delta_\gamma(x;x), \quad (180)$$

$$\xrightarrow{\text{IR}}-\frac{3}{4}\kappa^2\dot{H}a_0^4(t)\frac{-\dot{H}}{H^2(t)}\frac{1}{24\pi^2}\left(\frac{H_I^6-H^6(t)}{\dot{H}^2}\right), \quad (181)$$

$$=\frac{\kappa^2}{32\pi^2}\left(\frac{H_I^6-H^6(t)}{H^2(t)}\right)a_0^4(t). \quad (182)$$

In the notation used at the end of Sec. V there are contributions for $N=-2$ and $N=+4$ with coefficients of $\kappa^2/(32\pi^2)$ times $+H_I^6$ and -1 , respectively.

Tables I–III give the various cubic interaction vertices. Note that those of Table I have been partially symmetrized by making the external leg attach to pseudograviton No. 1. It should also be noted that derivatives with respect to the attached field are interpreted through integration by parts (IBP) as acting on the entire result. Further, spatial translation invariance (STI) allows the result to depend only upon the conformal time after the expectation value is taken. For example, in vertex No. 3 of Table I, the derivative with respect to line No. 1 is interpreted as follows:

$$\partial_1^{\alpha_2}\xrightarrow{\text{IBP}}-\frac{\partial}{\partial x_{\alpha_2}}\xrightarrow{\text{STI}}t^{\alpha_2}\frac{d}{d\eta}. \quad (183)$$

Because of the $\eta^{\alpha_1\beta_1}$ vertex No. 3 contributes to $\alpha(t)$ and $\gamma(t)$ with opposite signs. The contribution to $\alpha(t)$ is

$$\frac{d}{d\eta}\left\{ i\kappa\frac{i}{2}\kappa H\Omega^3\langle 0|T[\psi_{\alpha_2\beta_2}(x)\psi_{\alpha_3\beta_3}(x')]|0\rangle t^{\alpha_2}t^{\beta_2}\eta^{\alpha_3\beta_3}\right\}_{x'=x} \\ =\frac{d}{d\eta}\left\{ -\frac{1}{2}\kappa^2H\Omega^3i\Delta_\alpha(x;x')\right\}_{x'=x}, \quad (184)$$

$$\xrightarrow{\text{IR}}-3\kappa^2H^2(t)a_0^4(t)\frac{\dot{H}^2}{H^4(t)}\frac{1}{24\pi^2}\left(\frac{H_I^6-H^6(t)}{\dot{H}^2}\right), \quad (185)$$

$$=-\frac{\kappa^2}{8\pi^2}\left(\frac{H_I^6-H^6(t)}{H^2(t)}\right)a_0^4(t). \quad (186)$$

When the contributions from all vertices are summed the amputated one-point functions are

$$\alpha(t)\xrightarrow{\text{IR}}-\frac{3\kappa^2}{32\pi^2}\left(\frac{H_I^6-H^6(t)}{H^2(t)}\right)a_0^4(t), \quad (187)$$

$$\gamma(t)\xrightarrow{\text{IR}}+\frac{3\kappa^2}{32\pi^2}\left(\frac{H_I^6-H^6(t)}{H^2(t)}\right)a_0^4(t), \quad (188)$$

$$\delta(t)\xrightarrow{\text{IR}}-\frac{\kappa^2}{8\pi^2}\left(\frac{H_I^6-H^6(t)}{H^2(t)}\right)\left(\frac{\sqrt{-\dot{H}}}{H(t)}\right)a_0^4(t). \quad (189)$$

In the notation at the end of Sec. V this corresponds to the following coefficients:

$$\alpha_{-2} = -\gamma_{-2} = \frac{3}{4}\delta_{-2} = -\frac{3\kappa^2}{32\pi^2}H_I^6, \quad (190)$$

$$\alpha_{+4} = -\gamma_{+4} = \frac{3}{4}\delta_{+4} = \frac{3\kappa^2}{32\pi^2}. \quad (191)$$

From Eqs. (141),(142) we see that the cosmological expansion rate and the quantum-corrected scalar are

$$H_{\text{eff}}(\bar{t}) = H(\bar{t}) \times \left\{ 1 - \frac{\kappa^2}{72\pi^2} \left(\frac{H_I^6 - 3H_I^3 H^3(\bar{t}) + 2H^6(\bar{t})}{H^4(\bar{t})} \right) + \dots \right\}, \quad (192)$$

$$\Phi(\bar{t}) = \varphi_0(\bar{t}) \times \left\{ 1 + \frac{\kappa^2}{576\pi^2} \left(\frac{H_I^6 - 2H_I^3 H^3(\bar{t}) + H^6(\bar{t})}{H^4(\bar{t})} \right) + \dots \right\}. \quad (193)$$

It should be noted that without independent knowledge of the scalar potential there is no way of distinguishing classical and quantum contributions in $H_{\text{eff}}(\bar{t})$. Thus it is possible for a huge quantum back reaction to masquerade as a modification of the classical scalar potential. It should also be noted that quantum corrections *increase* $\Phi(\bar{t})$. Thus it is not possible to interpret the slowing of $H_{\text{eff}}(\bar{t})$ as the result of quantum corrections having pushed the scalar slightly down its classical potential.

VIII. SUMMARY AND DISCUSSION

This paper is first of all a check on the calculation of Mukhanov, Abramo, and Brandenberger. We certainly agree with the sign and the leading time dependence that can be inferred for $H_{\text{eff}}(\bar{t})$ and $\Phi(\bar{t})$ from their published results [1,2]. From unpublished work we see that even the numerical factors agree. It is worth emphasizing that we employed the standard formalism of covariant quantization while they used a truncated version of the canonical formalism. The two calculations were also done in completely different gauges: we added a covariant gauge fixing term whereas they used a

TABLE II. Vertex factors contracted into $\psi_{\alpha_1\beta_1}\bar{\omega}_{\alpha_2}\omega_{\alpha_3}$.

No.	Vertex factor	No.	Vertex factor
1	$-\kappa\Omega^2\eta^{\alpha_2(\alpha_1\eta^{\beta_1)\alpha_3}\partial_2\cdot\partial_3}$	6	$\frac{1}{2}\kappa\Omega^2\eta^{\alpha_1\beta_1}\partial_2^{\alpha_2}\partial_1^{\alpha_3}$
2	$-\kappa\Omega^2\eta^{\alpha_3(\alpha_1\partial_2^{\beta_1})\partial_3^{\alpha_2}}$	7	$-\kappa H\Omega^3\eta^{\alpha_1\beta_1}\partial_2^{\alpha_2}t^{\alpha_3}$
3	$-\kappa\Omega^2\eta^{\alpha_2(\alpha_1\partial_2^{\beta_1})\partial_1^{\alpha_3}}$	8	$-2\kappa H\Omega^3\eta^{\alpha_3(\alpha_1\partial_3^{\beta_1})}t^{\alpha_2}$
4	$2\kappa H\Omega^3\eta^{\alpha_2(\alpha_1\partial_2^{\beta_1})}t^{\alpha_3}$	9	$-\kappa H\Omega^3\eta^{\alpha_1\beta_1}\partial_1^{\alpha_3}t^{\alpha_2}$
5	$\kappa\Omega^2\eta^{\alpha_3(\alpha_1\partial_3^{\beta_1})\partial_2^{\alpha_2}}$	10	$2\kappa H^2\Omega^4\eta^{\alpha_1\beta_1}t^{\alpha_2}t^{\alpha_3}$

TABLE III. Cubic interactions involving ϕ .

No.	Interaction	No.	Interaction
1	$\frac{1}{8}\kappa^2\varphi_0'\Omega^2\phi'\psi^2$	6	$\frac{1}{2}\kappa\Omega^2\phi_{,\rho}\phi_{,\sigma}\psi^{\rho\sigma}$
2	$-\frac{1}{4}\kappa^2\varphi_0'\Omega^2\phi'\psi^{\rho\sigma}\psi_{\rho\sigma}$	7	$-\frac{1}{8}\kappa^2m^2\varphi_0\Omega^4\phi\psi^2$
3	$-\frac{1}{2}\kappa^2\varphi_0'\Omega^2\phi_{,\rho}\psi_0^{\rho}\psi$	8	$\frac{1}{4}\kappa^2m^2\varphi_0\Omega^4\phi\psi^{\rho\sigma}\psi_{\rho\sigma}$
4	$\kappa^2\varphi_0'\Omega^2\phi_{,\rho}\psi^{\rho\sigma}\psi_{\sigma 0}$	9	$-\frac{1}{4}\kappa m^2\Omega^4\phi^2\psi$
5	$-\frac{1}{4}\kappa\Omega^2\phi_{,\rho}\phi^{\rho}\psi$	10	$\kappa^2\varphi_0'\Omega^2\phi_{,\rho}\bar{\omega}_0\omega^{\rho}$

physical gauge. It would be difficult to imagine two more completely different calculational schemes. Yet we got the same results in the end, for both $H_{\text{eff}}(\bar{t})$ and $\Phi(\bar{t})$. This is an enormously powerful check on the validity of their work and on the physical reality of the effect. If this back-reaction is a gauge chimera it is a remarkably consistent one.

Our motivation for this work was the excellent questions posed earlier this year by Unruh [3]. Although our calculation is itself a sort of answer we have analyzed some of his arguments more generally in Sec. III. In particular, it turns out that the seeming disagreement between Unruh's long wavelength solutions and those of Mukhanov and co-workers derives from different definitions for what is "zeroth order" when expanding in powers of the wave number.⁹ If one keeps higher terms in k^2 it turns out that Unruh's second solution implies an order k^2 result for the variable used by Mukhanov and co-workers, and this result is just $4k^2/\kappa^2$ times their second solution. So neither of the long wavelength solutions of Mukhanov and co-workers is unphysical.

It is important to understand that a compelling physical mechanism underlies the back-reaction of Mukhanov and co-workers. It is the self-gravitation between superadiabatically amplified long wavelength modes. A simple physical model is that virtual particles whose wavelengths are comparable to the Hubble radius become trapped in the expansion of space-time and are not able to recombine. As the particles are pulled apart their long range gravitational potentials fill the intervening space, adding with the potentials of earlier pairs. Because gravitation is attractive these potentials resist the further expansion of spacetime, thereby slowing inflation. There is absolutely no question that this process should occur for quanta, such as gravitons and minimally coupled scalars, which lack conformal invariance but are still massless on the Hubble scale. The only issues concern the strength of the back-reaction, its time dependence, and whether or not it can eventually stop inflation.

The analogous back-reaction has already been demonstrated for gravitons when inflation is driven by a positive bare cosmological constant [7]. In this case it comes at two loops because the pair creation event is already one loop and

⁹After the completion of this work it was brought to our attention that Kodama and Hamazaki have already clarified the relation between Unruh's solutions and those of Mukhanov [17]. It should also be noted that Unruh has independently withdrawn the claim of a disagreement.

the absence of linearized mixing between the dynamical spin-2 gravitons and the spin-0 gravitational potentials postpones self-gravitation to next order. When the superadiabatically amplified quanta are themselves scalar their mixing with the spin-0 gravitational potentials allows self-gravitation to occur at one loop order. However, there does not have to be such a one loop effect [19]. The feature which seems to distinguish those scalar-driven models which show slowing at one loop from those which slow at two loops is the rate at which superadiabatic amplification injects 0-point energy. If this is less than or equal to the physical three-volume's inflation then there is no one loop effect; if superadiabatic amplification injects 0-point energy faster than the three-volume inflates to absorb it then there is a one loop effect.

Of course gravitons presumably drive a two loop effect in this model as well. There may also be significant scalar effects at higher loops. Higher loop processes are interesting in that they derive from the coherent superposition of interactions over the invariant volume of the past light cone, which can grow arbitrarily large. There is no barrier to considering such questions in the covariant formalism we have developed. The formalism of Mukhanov and co-workers would have to be extended to make this possible.

It may be of general interest that we were able to extend the Feynman rules of Iliopoulos *et al.* [4] so that they apply to an arbitrary scalar potential. This was the work of Sec. IV. Of course we can only express the propagators as mode sums, where even the mode functions remain to be determined. But we have shown in Sec. VI how these mode functions can be usefully expanded, both in the ultraviolet and in the infrared. And the calculation is an explicit example of how interesting effects can be obtained. It should now be possible to redo the two loop computation of Tsamis and Woodard [7] for an arbitrary background. This should completely determine the effective field equations needed to evolve past the end of inflation to arbitrarily late times [20].

ACKNOWLEDGMENTS

It is a pleasure to acknowledge stimulating and informative conversations with R. H. Brandenberger and V. F. Mukhanov. We are also grateful to the University of Crete for its hospitality at the inception of this project. This work was partially supported by the U. S. DOE Contract No. DE-FG02-97ER-41029, by NSF Grant No. 94092715, by NATO Grant No. CRG-971166, and by the Institute for Fundamental Theory.

-
- [1] V. Mukhanov, L. R. W. Abramo, and R. Brandenberger, *Phys. Rev. Lett.* **78**, 1624 (1997).
 - [2] L. R. W. Abramo, R. H. Brandenberger, and V. F. Mukhanov, *Phys. Rev. D* **56**, 3248 (1997).
 - [3] W. Unruh, "Cosmological Long Wavelength Perturbations," astro-ph/9802323.
 - [4] J. Iliopoulos, T. N. Tomaras, N. C. Tsamis, and R. P. Woodard, *Nucl. Phys.* **B534**, 419 (1998).
 - [5] J. Schwinger, *J. Math. Phys.* **2**, 407 (1961); *Particles, Sources and Fields* (Addison-Wesley, Reading, MA, 1970).
 - [6] A. D. Linde, *Phys. Lett.* **129B**, 177 (1983).
 - [7] N. C. Tsamis and R. P. Woodard, *Nucl. Phys.* **B474**, 235 (1996); *Ann. Phys. (N.Y.)* **253**, 1 (1997).
 - [8] L. P. Grishchuk, *Sov. Phys. JETP* **40**, 409 (1974); *Class. Quantum Grav.* **10**, 2449 (1993).
 - [9] F. Bloch and H. Nordsieck, *Phys. Rev.* **52**, 54 (1937).
 - [10] S. Weinberg, *Phys. Rev.* **140**, B516 (1965).
 - [11] G. Feinberg and J. Sucher, *Phys. Rev.* **166**, 1638 (1968).
 - [12] J. F. Donoghue, *Phys. Rev. D* **50**, 3874 (1994); *Phys. Rev. Lett.* **72**, 2996 (1994).
 - [13] T. Vachaspati and M. Trodden, "Causality and Cosmic Inflation," gr-qc/9811037.
 - [14] M. E. Peskin and D. V. Schroeder, *An Introduction to Quantum Field Theory* (Addison-Wesley, Reading, MA, 1995).
 - [15] B. S. DeWitt, in *Relativity Groups and Topology II*, edited by B. S. DeWitt and R. Stora (North-Holland, Amsterdam, 1984).
 - [16] V. Mukhanov, H. Feldman, and R. Brandenberger, *Phys. Rep.* **215**, 203 (1992).
 - [17] H. Kodama and T. Hamazaki, *Prog. Theor. Phys.* **96**, 949 (1996); *Phys. Rev. D* **57**, 7177 (1998).
 - [18] A. R. Liddle and D. H. Lyth, *Phys. Rep.* **231**, 1 (1993).
 - [19] L. R. Abramo and R. P. Woodard, *Phys. Rev. D* **60**, 044011 (1999).
 - [20] N. C. Tsamis and R. P. Woodard, *Ann. Phys. (N.Y.)* **267**, 145 (1998).